Can we study strongly stratified turbulence in a laboratory experiment?

+ focus on the vertical spectra...

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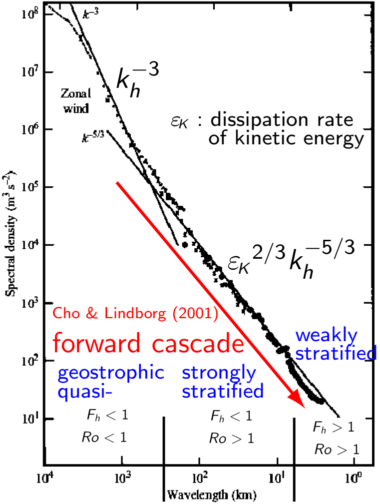
Paul Billant and Jean-Marc Chomaz

1st September 2015

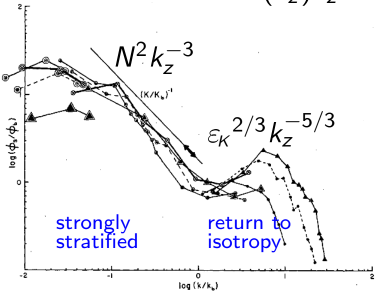
Anisotropic spectra in the atmosphere and oceans

horizontal spectrum $E(k_h)$



Gage & Nastrom (1986)

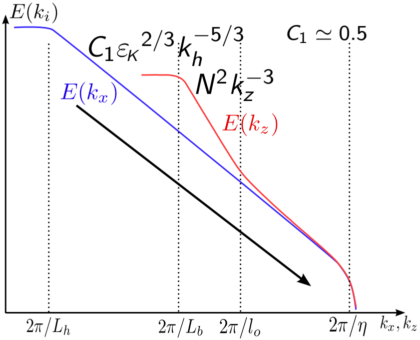
vertical spectrum of vertical shear $E(k_z)k_z^2$



Gargett *et al.* (1981)

Theory of "strongly stratified turbulence"

$F_h < 1$ and $Ro > 1$, (Lindborg, 2006)



Different scales:

- buoyancy length scale $L_b = U/N$
- Ozmidov length scale $l_o = (\varepsilon_K/N^3)^{1/2}$
  "the largest horizontal scale that can overturn" (Riley & Lindborg, 2008)
- Kolmogorov length scale η (dissipative structures)

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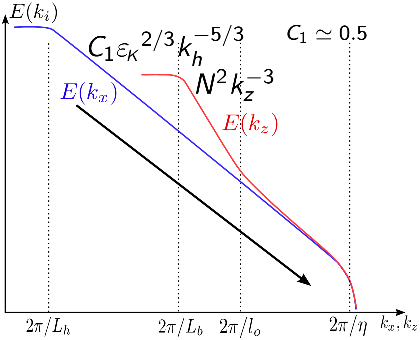

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viscous condition $l_o \gg \eta$ (Brethouwer *et al.*, 2007)

- turbulent buoyancy Reynolds number $\mathcal{R}_t = \left(\frac{l_o}{\eta}\right)^{4/3} = \frac{\varepsilon_K}{\nu N^2} \gg 1$
- buoyancy Reynolds number $\mathcal{R} = ReF_h{}^2 \propto \mathcal{R}_t \gg 1$

Three issues about strongly stratified turbulence

- No experimental validation.
- “Small Froude number and large buoyancy Reynolds number”... How? Definitions?
- Vertical spectra consistent with measurements in the atmosphere and the oceans?

Why no experimental validation?

- With salt, $\max(N) \sim 1$ rad/s.
  Strongly stratified implies $U/L < N$, i.e. slow or very large...
  Thus the condition $\mathcal{R} = Re{F_h}^2 \gg 1$ is difficult to fulfill.

- Inhomogenous optical index + turbulent mixing $\Rightarrow$ blurry.

  Optical measurements become difficult when it becomes interesting!

- Decaying vs forced turbulence...

Different configurations...

Numerics

Forced

- Only small wavenumbers, with $k_z = 0$ or not...
- Only vortices, only waves or both...
- Statistically homogeneous and stationary.

Decaying

- Isotropic turbulence (waves and vortices),
- Large-scale flows,
  - Waves or vortices,
  - $F_h > 1$ or $F_h < 1$.

Experiments

(solid objects and boundary layers)

Forced

- Generators of large-scale flows (waves and/or vortices),
- Isolated structures or interacting structures.

Decaying

Wakes of objects:

- shape (grid, array of vertical pens)
- $F_h > 1$ or $F_h < 1$.

Outlines

1. Experimental study of a forced stratified turbulent-like flow

   Augier, Billant, Negretti & Chomaz (2014)

2. Numerical study of strongly stratified turbulence

   Augier, Billant & Chomaz (2015)

3. What can we get in the Coriolis platform?

4. Back to vertical spectra...

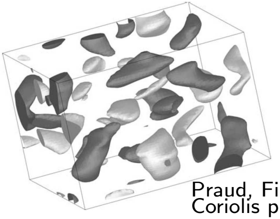
Experiment: forced stratified turbulence

Previous experiments of turbulence in stratified fluids

- $\mathcal{R}_t \lesssim 1$
- decaying turbulence

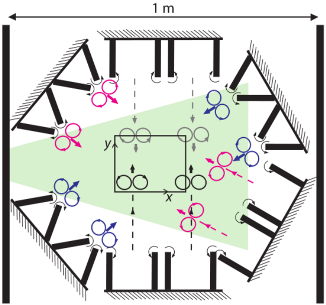
$\Rightarrow$ viscous regime

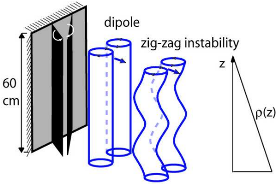
ω_z

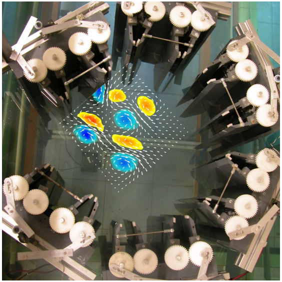


Praud, Fincham & Sommeria, (2005)
Coriolis platform

Experiment: forced stratified turbulence



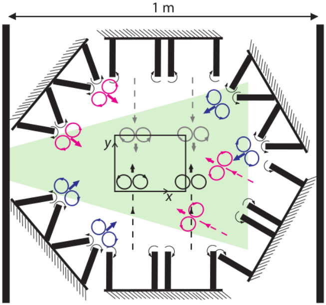


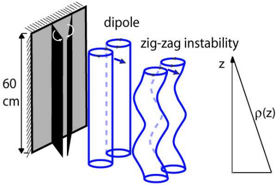


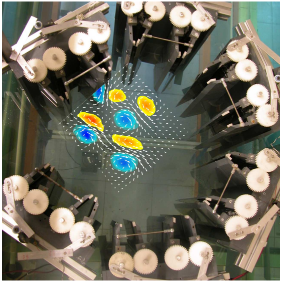
- forced by vertically invariant vortical modes, no vertical length scale imposed ($k_z = 0$).
- tank $2 \times 1\text{m}^2 \times 60\text{cm}$ with a linear profile $N \simeq 1.7$ rad/s
- measurements by particle image velocimetry (PIV)

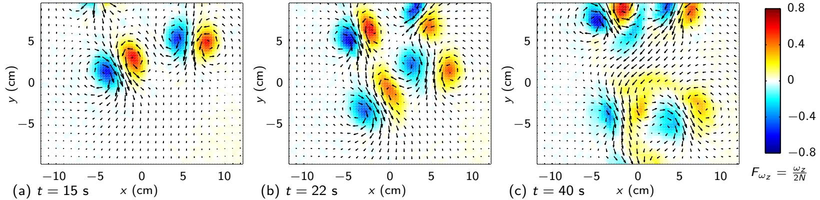
  horizontal and vertical cross-sections

Experiment: forced stratified turbulence



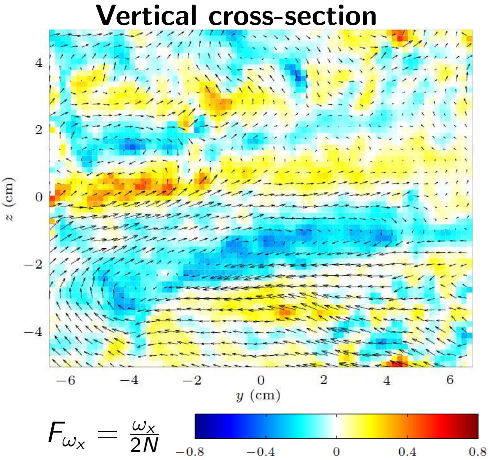






(a) $t = 15$ s (b) $t = 22$ s (c) $t = 40$ s

Experiments: turbulent-like flow, $\mathcal{R} = 310$



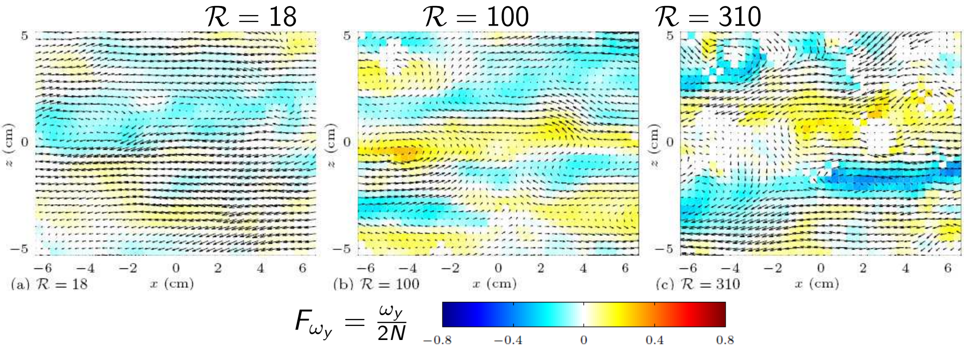
$F_{\omega_x} = \frac{\omega_x}{2N}$

- anisotropy (strong vertical shear)
- $\omega_y/(2N) \simeq 1$ shows that $Ri \simeq 0.25$
- secondary instabilities and overturnings

Experiments: variation of the forcing

effect of the buoyancy Reynolds number $\mathcal{R} = ReF_h{}^2$

vertical cross-sections



transition from viscous to inviscid regime when $\mathcal{R}$ is increased

From studies on the non-linear evolution of the zigzag instabilities (Deloncle, Billant & Chomaz, 2008)

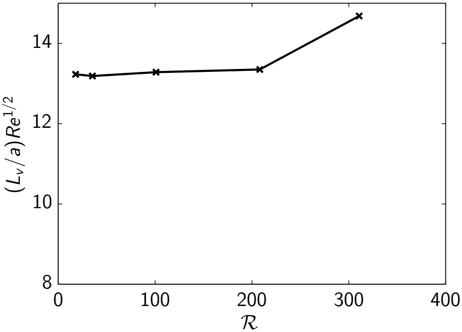
For one dipole, transition to turbulence if $\mathcal{R} > \mathcal{R}_c = 340$.

Experiments: vertical Taylor microscale $L_v = (2\langle {u_x}^2\rangle / \langle[\partial_z u_x]^2\rangle)$

effect of the buoyancy Reynolds number $\mathcal{R} = ReF_h{}^2$

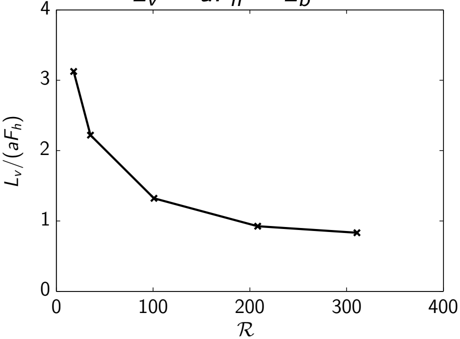
Viscous scaling law:

$L_v \sim aRe^{-1/2}$



Inviscid scaling law:

$L_v \sim aF_h \sim L_b$

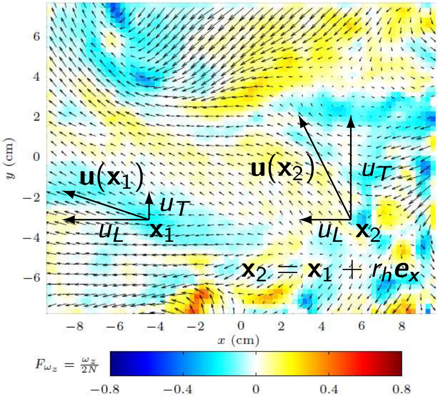


transition from viscous to inviscid scaling law when $\mathcal{R}$ is increased

Experiments: energy distribution

horizontal structure functions

Horizontal cross-section



Definition

- transverse structure function
  $S_{2T}(r_h) = \langle [u_T(\mathbf{x}_2) - u_T(\mathbf{x}_1)]^2 \rangle$
- longitudinal structure function
  $S_{2L}(r_h) = \langle [u_L(\mathbf{x}_2) - u_L(\mathbf{x}_1)]^2 \rangle$

horizontal spectrum $E(k_h)$ linked to the Fourier transform of S_2

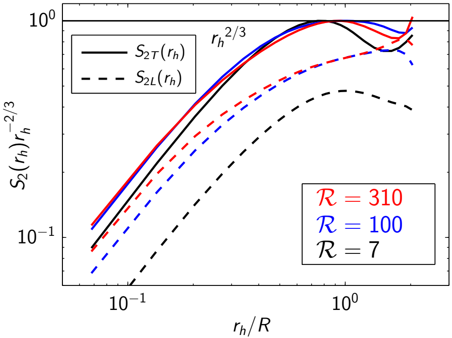
Fully turbulent flow:

- spectrum $\propto k_h^{-5/3}$
- structure functions $\propto r_h^{2/3}$

Experiments: horizontal second order structure functions

compensated horizontal second order structure functions

transverse S_{2T} and longitudinal S_{2L}



- no $r_h^{2/3}$ (inertial range)

but when $\mathcal{R}$ is increased

- more energy at small scales
- more increase for S_{2L}

  (secondary instabilities)

close to the transition to the strongly stratified turbulent regime

but $\mathcal{R}$ still not large enough... viscosity influences the flow

Conclusion experiments

Novel experience of maintained stratified turbulence, $\mathcal{R} \sim 1$

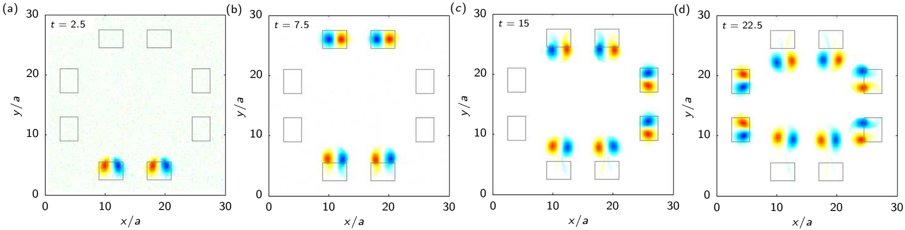
- close to the transition to the strongly stratified turbulent regime (strong shear, $F_v \sim 1$, secondary instabilities)
- but limited parameter range ($\mathcal{R}$ and F_h): no inertial range
- difficult measurements (we need homogeneous refractive index to improve the quality of the velocity fields).
- very particular forcing, only with dipoles...

Unfortunately: cannot increase the size of the vortices just by increasing the size of the vortex generators!

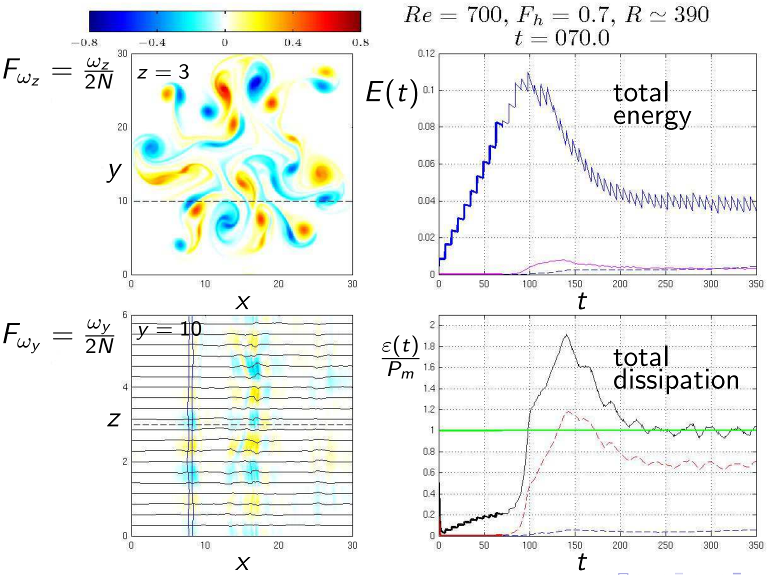
Numerical simulations

- Navier-Stokes solver (Boussinesq approximation):

  pseudo-spectral code, MPI parallel computing, from $256 \times 256 \times 128$ to $1600 \times 1600 \times 320$
- DNS with forcing similar to the experiments
  - in physical space with columnar dipoles (Lamb-Oseen)
  - periodic in time

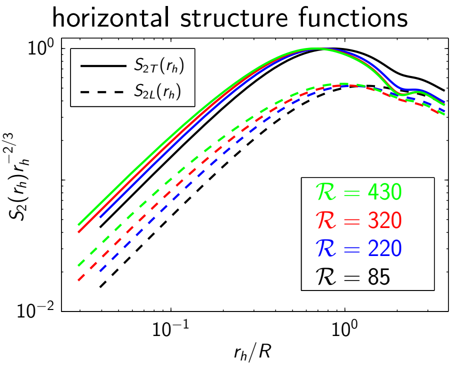


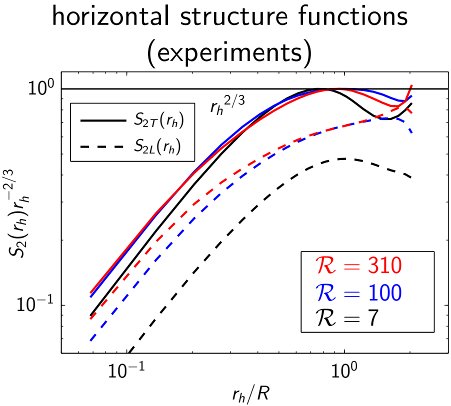
Time evolution $Re = 700$, $F_h = 0.7$, $\mathcal{R} = 390$



Statistical analysis of the stationary flows

Numerics: experimental parameters, $F_h = 0.85$





- no ${r_h}^{2/3}$ (inertial range)

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- more energy at small scales

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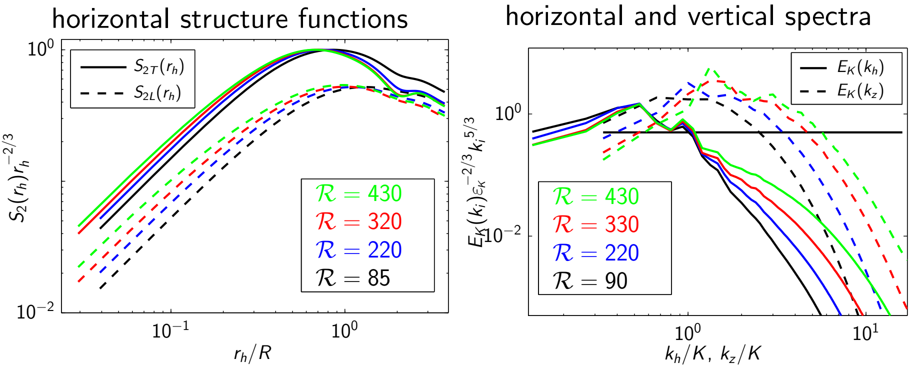

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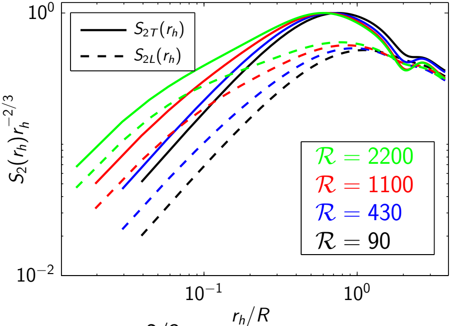
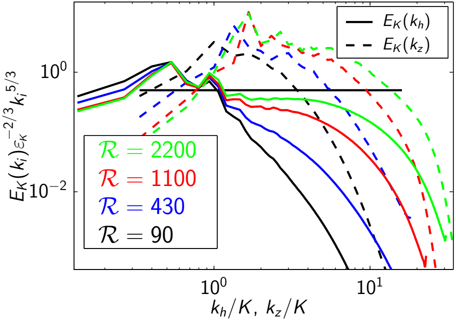
Statistical analysis of the stationary flows

Numerics: extension to moderate $\mathcal{R}$, $F_h = 0.85$

horizontal structure functions



horizontal and vertical spectra



- no ${r_h}^{2/3}$ (inertial range)

but when $\mathcal{R}$ is increased

- more energy at small scales

when $\mathcal{R}$ is increased:

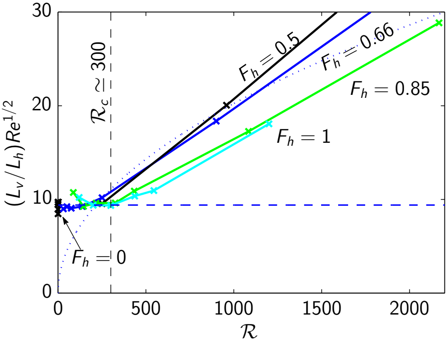
- ${k_h}^{-5/3}$ (inertial range)

Inertial range more visible with the spectra

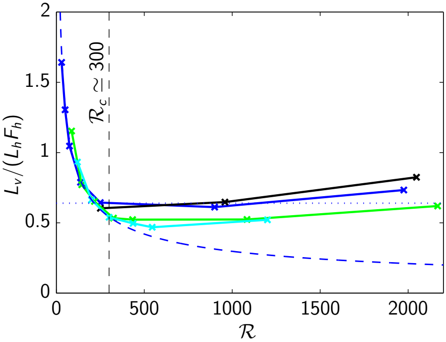
Characteristic aspect ratio L_v/L_h (Taylor microscales)

Numerics: extension to moderate $\mathcal{R}$ and small F_h

Viscous scaling law:
$L_v/L_h \sim Re^{-1/2}$

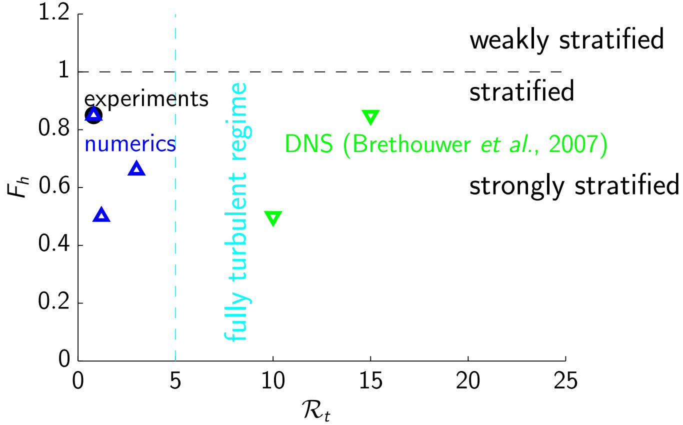


Inviscid scaling law:
$L_v/L_h \sim F_h$

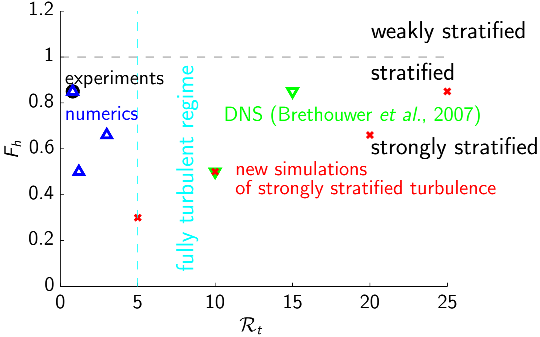


transition from viscous to inviscid scaling law when $\mathcal{R}$ is increased

$[\mathcal{R}_t,\ F_h]$ space



$[\mathcal{R}_t,\ F_h]$ space



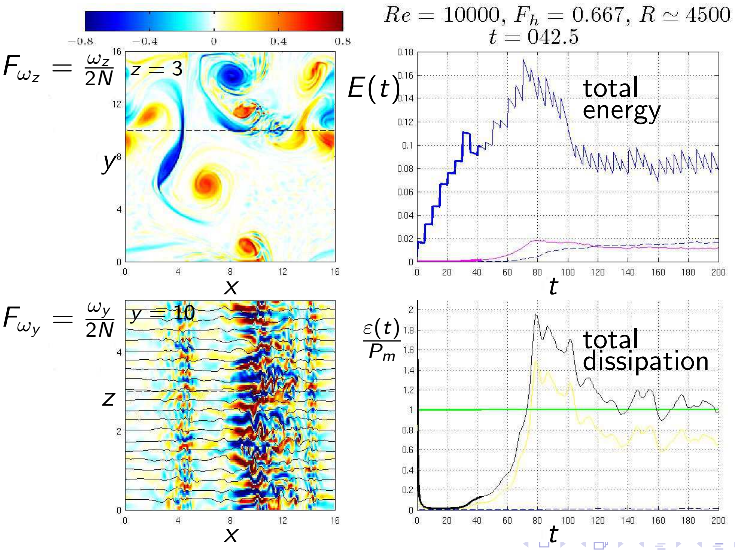
Large simulations of forced strongly stratified turbulence

quasi-DNS with weak hyperviscosity (Kolmogorov length scale nearly resolved)

F_h	Re	$\mathcal{R}$	F_{ht}	$\mathcal{R}_t$	$\mathcal{L}_h^2 \times \mathcal{L}_z$	$N_h^2 \times N_z$
0.29	28000	2285	0.0076	4.8	$16^2 \times 2.29$	$1792^2 \times 256$
0.50	22500	5625	0.013	12	$16^2 \times 4.00$	$1024^2 \times 256$
0.66	22500	10000	0.019	23	$16^2 \times 5.33$	$1152^2 \times 384$
0.85	20000	14610	0.021	32	$16^2 \times 6.86$	$896^2 \times 384$

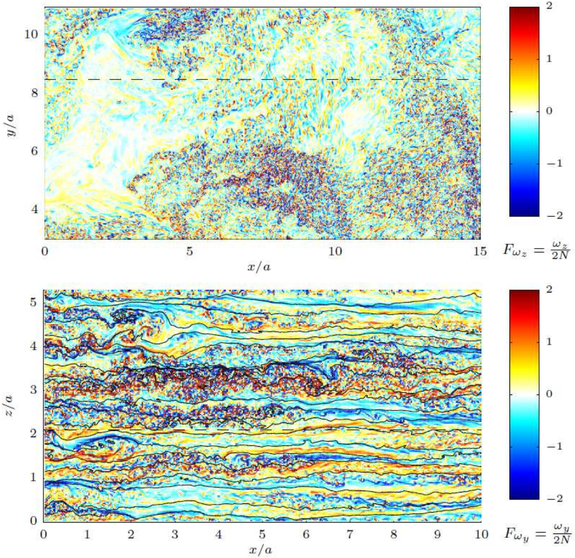
Time evolution $Re = 10000$, $F_h = 0.66$, $\mathcal{R} = 4500$

dipoles are periodically produced at a random location

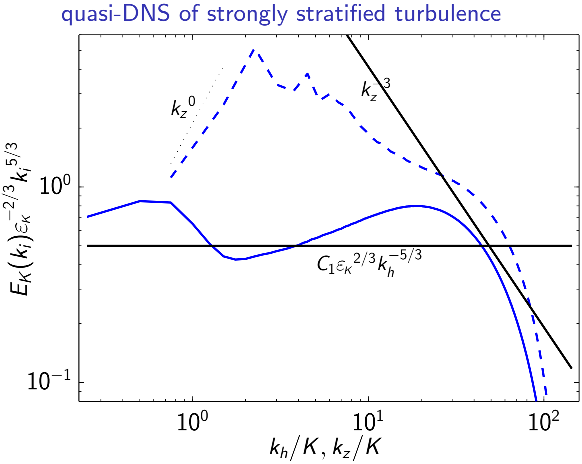


Cross-sections for $F_h = 0.66$ and $\mathcal{R}_t = 23$

quasi-DNS of strongly stratified turbulence



Horizontal and vertical spectra for $F_h = 0.66$ and $\mathcal{R}_t = 23$



spectra $\sim$ in agreement with the theory of strongly stratified turbulence

Conclusions DNS and quasi-DNS of strongly stratified turbulent flows forced by columnar dipoles

- Forcing similar to the experiments
  - reproduction of experimental results
  - extension $F_h \ll 1$ and $\mathcal{R}_t > 1$

We were at the "strongly stratified transition" in the experiments.

- We reach the strongly stratified turbulent regime and get similar results as in more classical turbulent simulations.

- Thresholds for strongly stratified turbulence (?)

$$F_h \simeq 0.8 \text{ and } \mathcal{R} \simeq 10000$$
$$F_{ht} \simeq 0.02 \text{ and } \mathcal{R}_t \simeq 10$$

- Two types of forcing: importance of inhomogeneity and non-stationarity...
  The same structural buoyancy Reynolds number can produce different values of the turbulent buoyancy Reynolds number.

What can we get in the Coriolis platform?

13 m diameter

What can we get in the Coriolis platform?

Increasing the size is efficient

Comparison with the experiment at LadHyX (∼1 m)

- We stay "strongly stratified": $F_h < 1$
  For simplicity, same Froude number as in the experiment at LadHyX for $F_h = 0.85$, $\mathcal{R} = 300$.

$$F_h = \frac{U}{L_h N} = \text{constant} \Rightarrow U \propto L_h$$

- Effect on the buoyancy Reynolds number

$$\mathcal{R} = ReF_h{}^2 \propto Re \propto L_h{}^2$$

LadHyX (∼1 m)	Coriolis (∼13 m)
$a \simeq 2$ cm	$a \simeq 20$ cm
$\mathcal{R} = 300$	$\mathcal{R} \simeq 30000$

Typically like the simulation for $F_h = 0.66$ and $\mathcal{R} = 4500$ ($\mathcal{R}_t \simeq 10$)

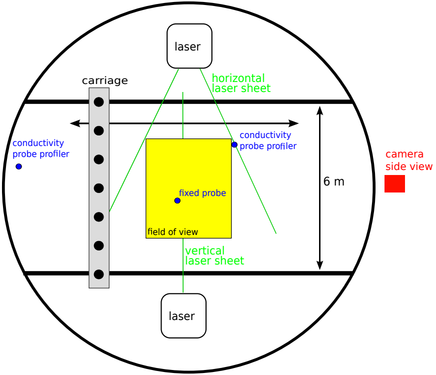
What can we get in the Coriolis platform?

How can we force large columnar vortices?

Can not upscale the vortex by upscaling the vortex generators

Wake of large vertically invariant objects, for example cylinders

EuHIT, with Erik Lindborg and Joel Sommeria

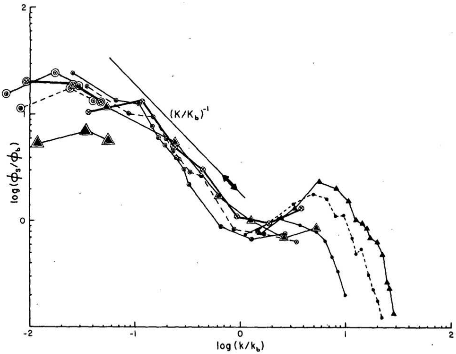


Rmq: decaying or “forced” turbulence...

Oceanic vertical spectra

data taken from Gargett *et al* (1981)

$$\frac{2{k_z}^2 E_{Kh}(k_z)}{\sqrt{\varepsilon_K N}}$$



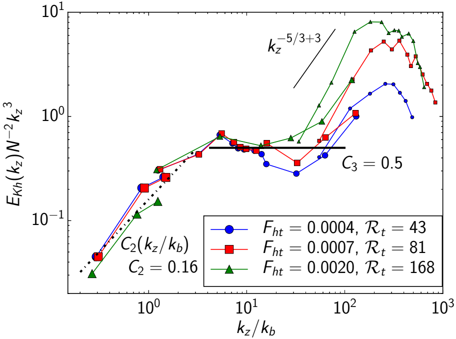
$$k_z/k_o$$

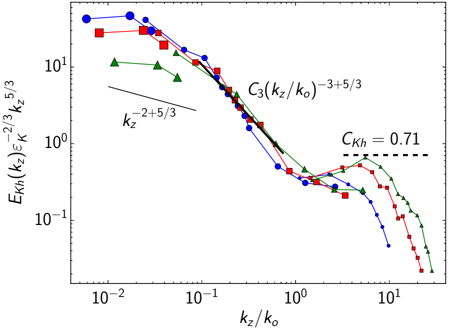
"Garret-Munk internal wave spectra":

- ${k_z}^{-2}$, ${k_z}^{-3}$ (thought to be due to internal gravity waves)
- ${k_z}^{-5/3}$ (thought to be due to turbulence).

Oceanic vertical spectra

data taken from Gargett *et al* (1981)



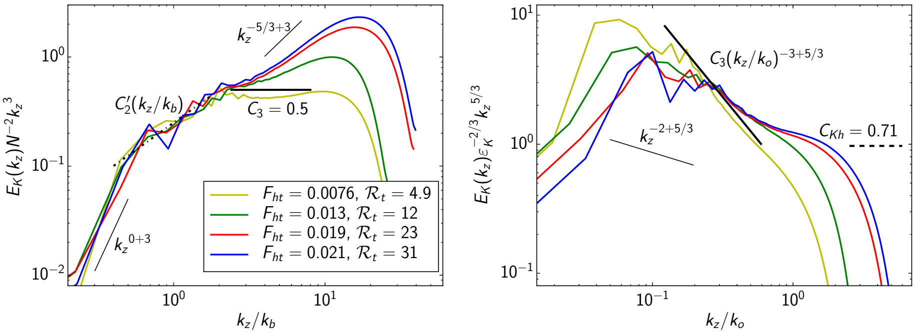


Two alternative representations showing three scaling laws:

- $E(k_z) = C_2 U_h N k_z^{-2}$, with $C_2 \simeq 0.16$,
- $E(k_z) = C_3 N^2 k_z^{-3}$, with $C_3 \simeq 0.5$,
- $E(k_z) = C_{Kh}\varepsilon_K{}^{2/3} k_z{}^{-5/3}$, with $C_{Kh} = 0.71$ the Kolmogorov constant for the vertical spectrum of horizontal kinetic energy.

Vertical spectra

data taken from Augier, Billant & Chomaz (2015)

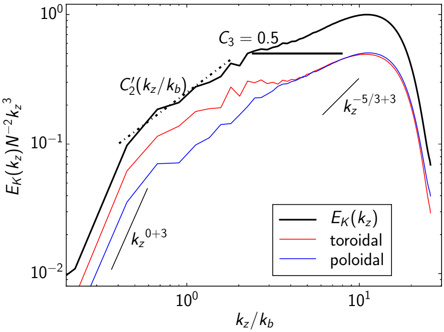


We can see the three scaling laws $U_h N k_z^{-2}$, $N^2 k_z^{-3}$ and $\varepsilon_K{}^{2/3} k_z{}^{-5/3}$ observed in the oceanic spectra!

Beware: here $C_2' = 0.25$, i.e. approximately twice the value for the oceanic spectra.

Vertical spectra: toroidal-poloidal decomposition

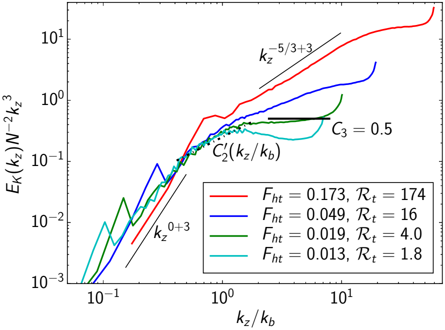
data taken from Augier, Billant & Chomaz (2015), for $F_{ht} = 0.013$ and $\mathcal{R}_t = 12$

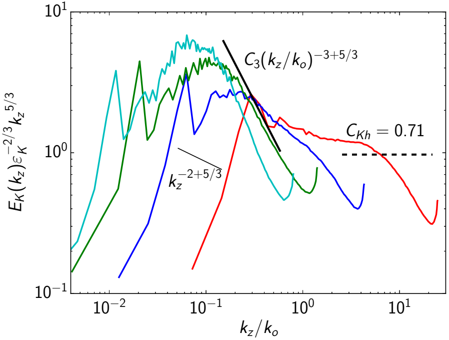


The k_z^{-2} and k_z^{-3} are dominated by vortices, meaning that they are not at all due to internal gravity waves!

Vertical spectra

data taken from Kimura & Herring (2012)





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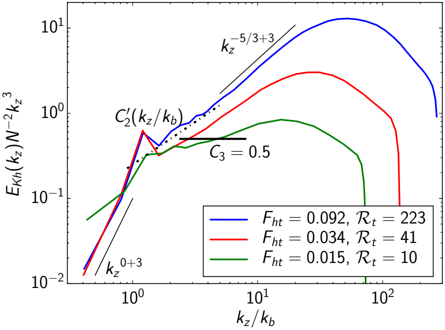
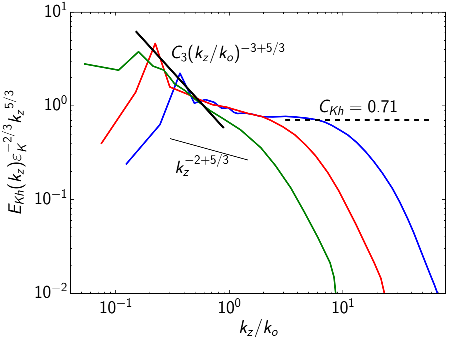
Vertical spectra

data taken from Almalkie & de Bruyn Kops (2012)





Beware: here $C_2' = 0.25$, i.e. approximately twice the value for the oceanic spectra.

Conclusions: vertical spectra

- 2 alternative representations two clearly show the three scaling laws
  - $E(k_z)N^{-2}{k_z}^3$ versus k_z/k_b,
  - $E(k_z)\varepsilon_K^{-2/3}{k_z}^{5/3}$ versus k_z/k_o,
- 1 new (?) scaling law $E(k_z) \sim U_h N k_z^{-2}$
- In the simulations, ${k_z}^{-2}$ and ${k_z}^{-3}$ are NOT due to waves.

Two small remarks...

- In articles, always provide F_{ht} and $\mathcal{R}_t$ (?)
- We could be more efficient by going towards open science (open software, scripts, data...)