Mixing and spatio-temporal analysis of stratified turbulence forced in rotational or divergent modes

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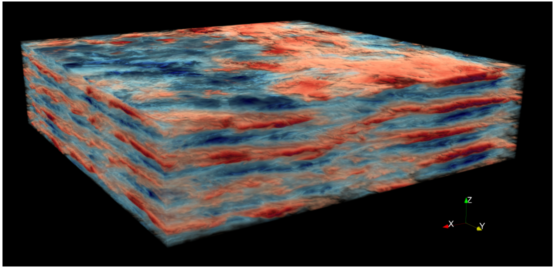


Model: Boussinesq equations

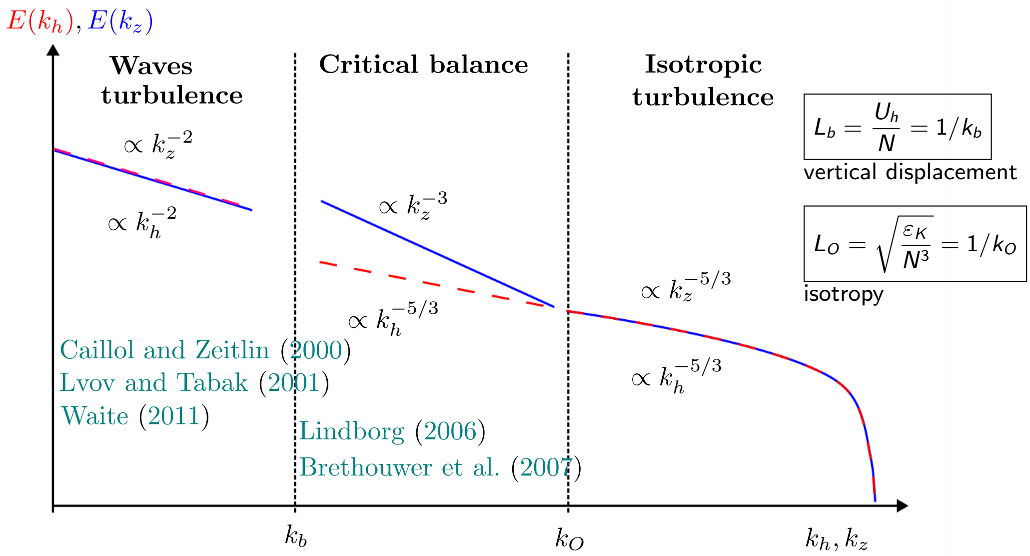
$$\nabla \cdot \mathbf{v} = 0,$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + b\mathbf{e}_z + \nu \Delta \mathbf{v} + \mathbf{F}_{\mathbf{v}},$$

$$\frac{\partial b}{\partial t} + \mathbf{v} \cdot \nabla b = -N^2 v_z - \kappa \Delta b + \mathbf{F}_{\mathbf{b}},$$

$\mathbf{v} = (v_x, v_y, v_z)$: velocity
p: total pressure
b: buoyancy
ν: viscosity
N: Brunt–Väisälä frequency
κ: Diffusivity
$\mathbf{F}_{\mathbf{v}}$, $\mathbf{F}_b$: forcings

Predictions: spectra for strongly stratified turbulence



Motivations

How the control parameters impact the flow?

How waves and eddies impact the flow?

How the forcing impacts the flow?

$\Rightarrow$ Numerical simulations in 3 different configurations. Soon available.

Flow decomposition

$k_h \neq 0$:

Vortical modes:

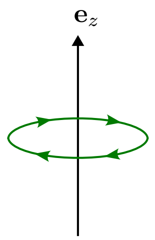
$$a_{\mathbf{k}}^{(0)} = \hat{v}_{t\mathbf{k}}$$

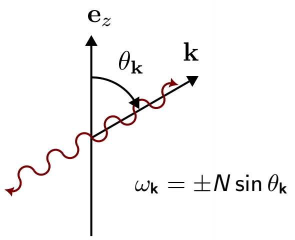
toroidal

Waves modes:

$$a_{\mathbf{k}}^{(\pm)} = \frac{1}{2}\left(\hat{v}_{p\mathbf{k}} \pm i\,\frac{\hat{b}_{\mathbf{k}}}{N}\right)$$

poloidal

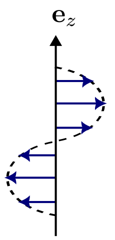




$k_h = 0$:

Shear modes:

$$a_{\mathbf{k}}^{(s)} = \left\{ \hat{\mathbf{v}}_{h\mathbf{k}}, \hat{b}_{\mathbf{k}} \mid k_h = 0 \right\}$$



Numerical simulations

Time correlated, anisotropic forcing at large scale $(k_f \sim 1)$:

$$\{k_{f,min} \le |k| \le k_{f,max}, \qquad |\theta_{\mathbf{k}} - \theta_f| \le \delta\theta_f/2\}$$

Constant kinetic energy injection $(P_K = 1)$.

3 datasets:

1) Toroidal forcing;
2) Poloidal forcing;
3) Projection $(\hat{v}_{t\mathbf{k}} = 0)$;

$\Rightarrow \quad \ge 90$ simulations with $n_h \times n_z = 640 \times 80 \quad \rightarrow 2560 \times 640$.

Shear modes are removed.

$$Pr \equiv \frac{\nu}{\kappa} = 1, \qquad Pr_4 \equiv \frac{\nu_4}{\kappa_4} = 1.$$

Numerical simulations

Control parameters: N ν $P_K = 1$

$\Rightarrow$ **Responses** (in steady state): $U_h \equiv rms(\mathbf{v}_h)$ ε_K ε_A

Horizontal Froude:

$$\boxed{F_h = \varepsilon_K / ({U_h}^2 N)}$$

Buoyancy Reynolds:

$$\boxed{\mathcal{R} = \varepsilon_K / (\nu N^2)}$$

Mixing ratio Caulfield (2020):

$$\boxed{\Gamma = \frac{\varepsilon_A}{\varepsilon_K}}$$

Open source code Fluidsim Mohanan et al. (2019):
$\rightarrow$ Pseudo spectral solver *ns3d.strat*.

Supercomputers:
Licallo, Azzurra, LEGI's cluster, Jean-Zay (GENCI), and Occigen (GENCI).

Results: toroidal forcing (dataset 1)

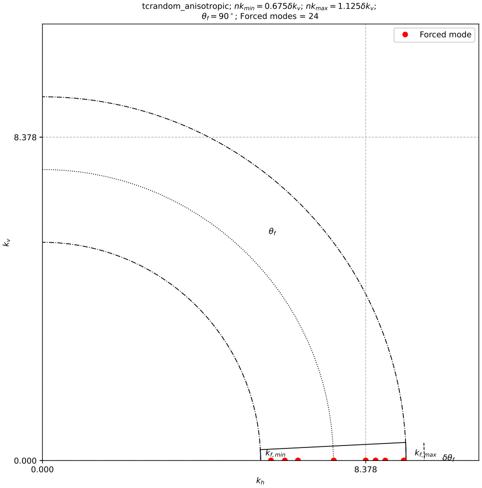
Forcing scheme:

- Forcing on $\hat{v}_{t\mathbf{k}}$ (vertical vorticity) Waite and Bartello (2004).

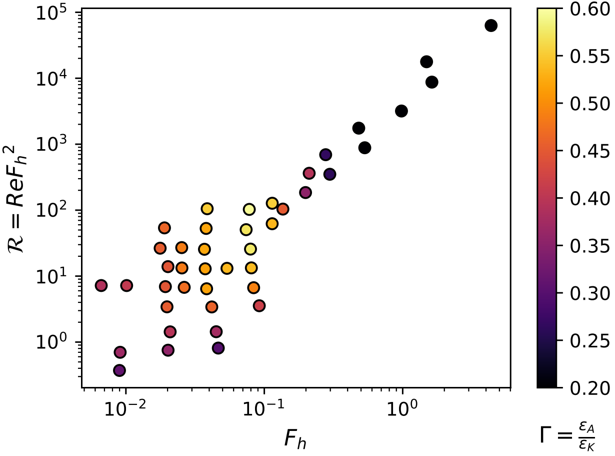
- On modes:

$$\{3 \leq k_h/\delta k_h \leq 5, \quad k_z = 0\}$$

("Columnar large vortices").



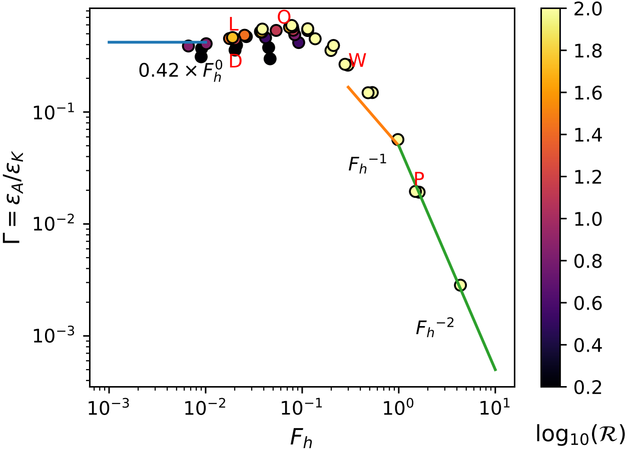
Results: toroidal forcing (dataset 1)



Mixing ratio:

- Does not depend on $\mathcal{R}$ if $\mathcal{R} \gg 1$ Portwood et al. (2019).

Results: toroidal forcing (dataset 1)

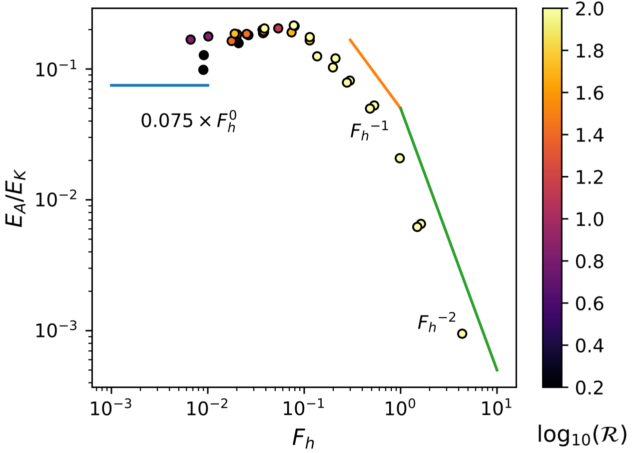


Mixing ratio:

- 5 regimes Brethouwer et al. (2007); Maffioli et al. (2016); Garanaik and Venayagamoorthy (2019):
  - LAST (L);
  - Strongly stratified, viscosity affected (D);
  - Optimal mixing (O);
  - Weakly stratified (W);
  - Passive scalar (P).

- $\lim_{F_h \to 0} \Gamma(F_h) \simeq 0.42$

Results: toroidal forcing (dataset 1)

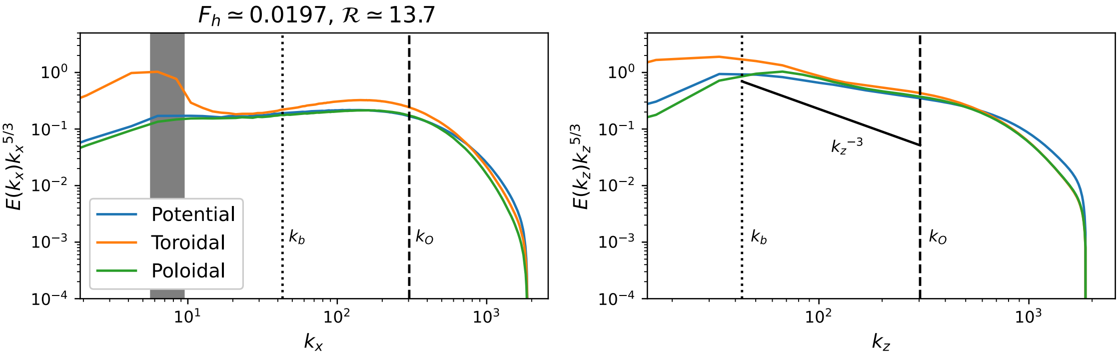


Mixing ratio:

- Optimal mixing when E_A/E_K is maximal Maffioli et al. (2016).

Results: toroidal forcing (dataset 1)

Spectra: LAST



- $E_{polo} \simeq E_{toro} \simeq E_{pot}$

- Bottleneck at k_b Augier et al. (2015)

- $E(k_z) \sim k_z^{-\alpha}$ for $k_b \leq k_z \leq k_O$ Lindborg (2006); Maffioli (2017)

Results: poloidal forcing (dataset 2)

Forcing scheme:

- Forcing on $\hat{v}_{p\mathbf{k}}$ (v_z and horizontally divergent).

- Forced modes:

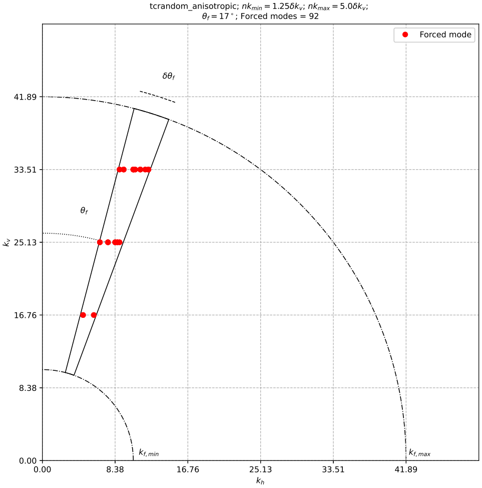
$$\{1.25 \leq |k|/\delta k_z \leq 5, \quad |\theta_{\mathbf{k}} - \theta_f| \leq \delta\theta_f\}$$

with $\theta_f = 0.3$ and $\delta\theta_f = 0.1$.

- Correlation time:

$$T_c = \frac{2\pi}{N \sin\theta_f}.$$

$\Rightarrow \quad \sim$ Forcing slow internal waves Waite and Bartello (2006).



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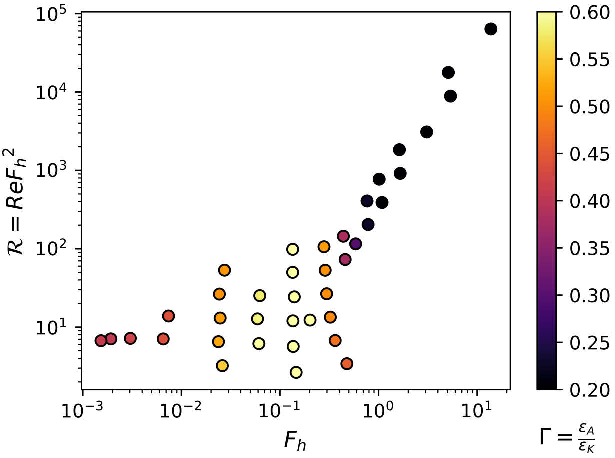

Mixing ratio:

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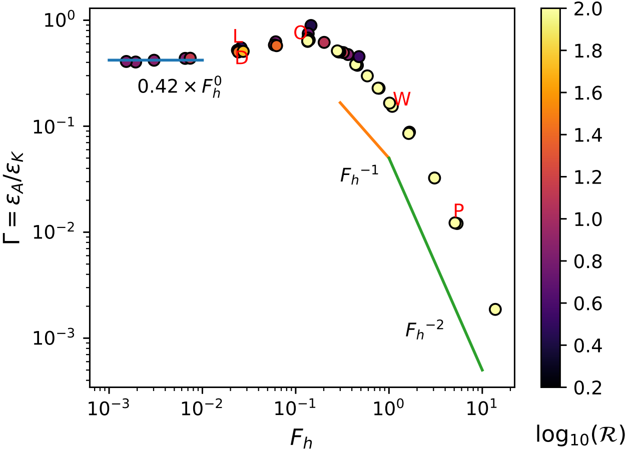
Results: poloidal forcing (dataset 2)



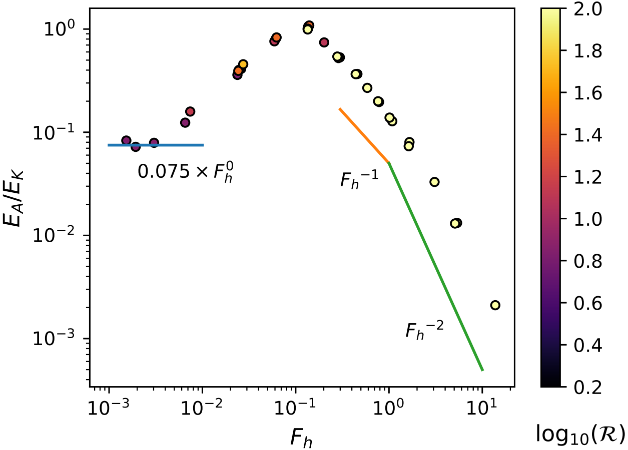
Mixing ratio:

- 4 regimes:
  - LAST (L);
  - Optimal mixing (O);
  - Weakly stratified (W);
  - Passive scalar (P).

No viscosity affected regime (D) because no simulations at $\mathcal{R} < 1$.

- $\lim_{F_h \to 0} \Gamma(F_h) \simeq 0.42$ ($\sim$ toroidal forcing).

Results: poloidal forcing (dataset 2)

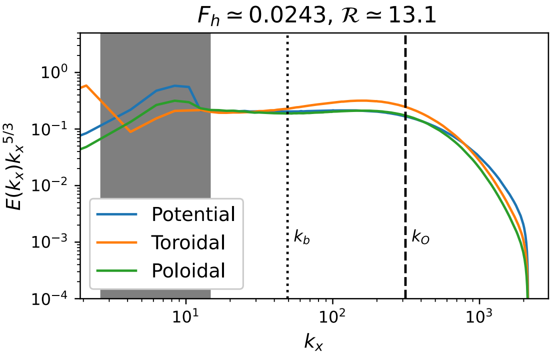


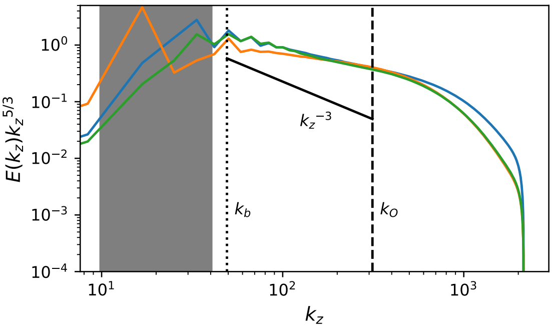
Mixing ratio:

- Optimal mixing when E_A/E_K is maximal.

Results: poloidal forcing (dataset 2)

Spectra: LAST





- $E_{polo} \simeq E_{toro} \simeq E_{pot}$
- $E(k_h = \delta k_h)$ grows with N
- Bottleneck at k_b Augier et al. (2015)
- $E(k_z) \sim k_z^{-\alpha}$ for $k_b \leq k_z \leq k_O$ Lindborg (2006); Maffioli (2017)

Results: poloidal projection (dataset 3)

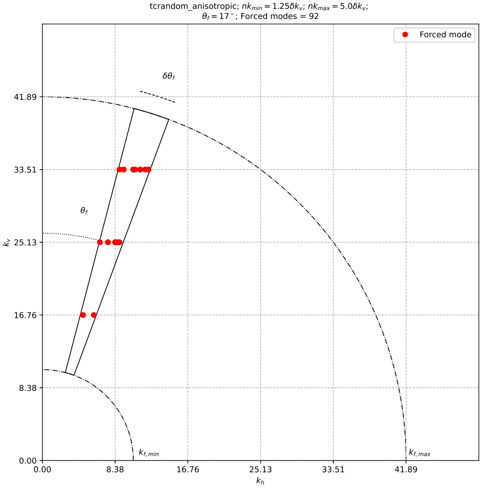
Forcing scheme:

Same as in dataset 2.

With projection:

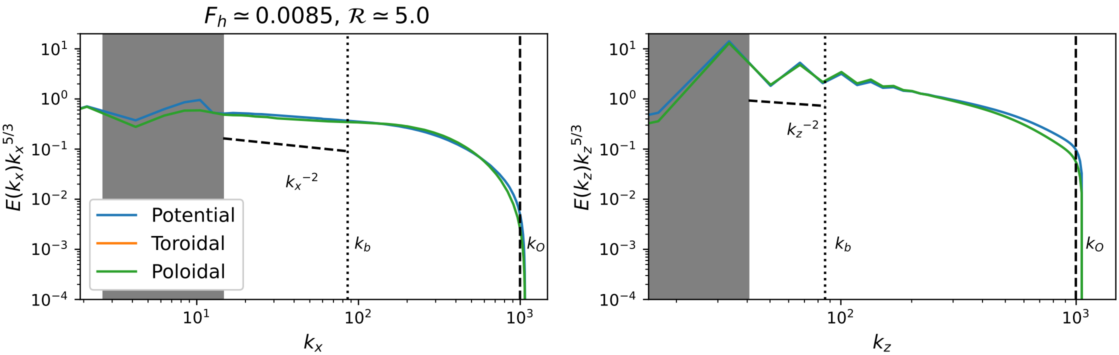
$$\boxed{\hat{v}_{t\mathbf{k}} = 0}$$

$\Rightarrow$ **Equations are changed to test internal gravity waves turbulence** (without shear modes and vortical modes).



Results: poloidal projection (dataset 3)

Spectra:



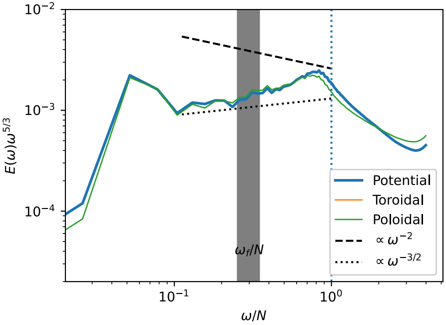
- $E_{polo} \simeq E_{pot}$

- $E(k_h = \delta k_h)$ grows with N

- $E(k) \sim k^{-2}$ for $k \leq k_b$ Caillol and Zeitlin (2000); Lvov and Tabak (2001)

Results: poloidal projection (dataset 3)

Spectra:



- $E_{polo} \simeq E_{pot}$

- $E(k_h = \delta k_h)$ grows with N

- $E(\omega) \sim \omega^{-3/2}$ for $\omega \ll N$ Caillol and Zeitlin (2000); Lvov and Tabak (2001)

Conclusion and perspectives

- **3 DNS datasets** without shear modes at various $(F_h, \mathcal{R})$:
  - Toroidal (vertical vorticity) forcing;
  - Poloidal (waves) forcing;
  - Poloidal projection.

- **5 regimes** Brethouwer et al. (2007); Maffioli et al. (2016); Garanaik and Venayagamoorthy (2019) including LAST and Weakly stratified.

- **Mixing coefficient** Caulfield (2020):
  - $\lim_{F_h \to 0} \Gamma \simeq 0.42$ for poloidal and toroidal forcings.
  - Optimal mixing when E_A/E_K is maximal.

- **Strongly stratified pure-poloidal regime**:
Compatible with $E(k) \sim k^{-2}$ and $E(\omega) \sim \omega^{-3/2}$ Caillol and Zeitlin (2000); Lvov and Tabak (2001).

Thank you for your attention

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