2D numerical simulations of stratified turbulence forced by internal waves

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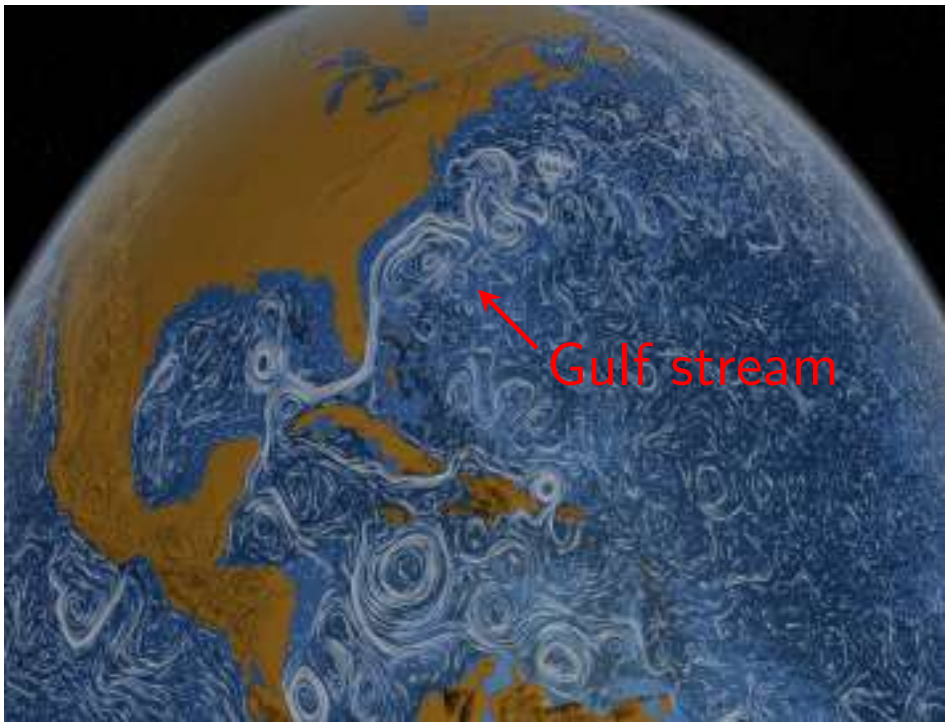
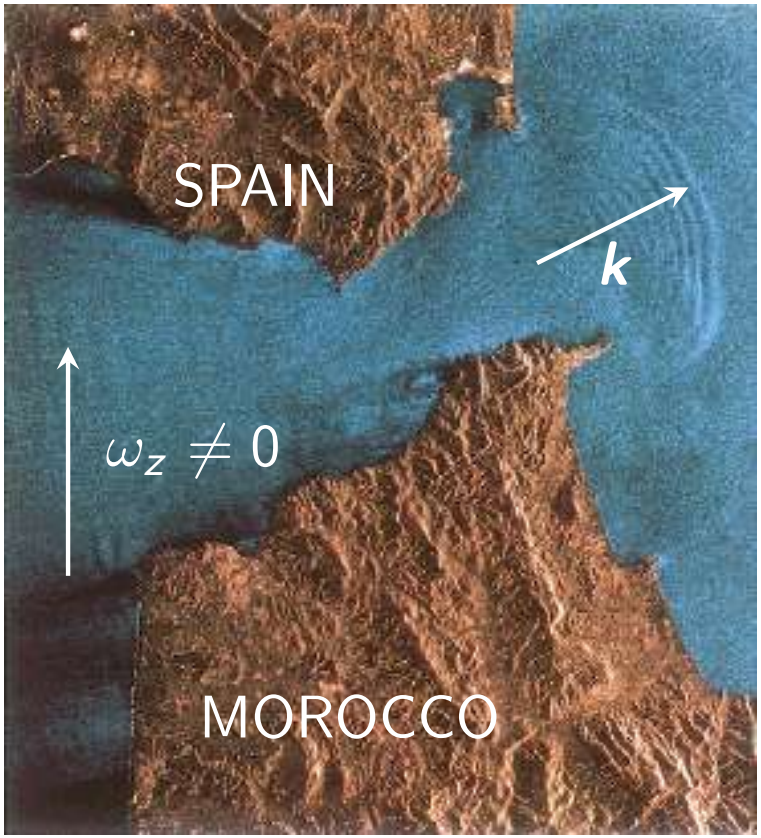
Oceans & Earth's climate system

Oceans are turbulent & stratified



https://www.nasa.gov

VORTICES

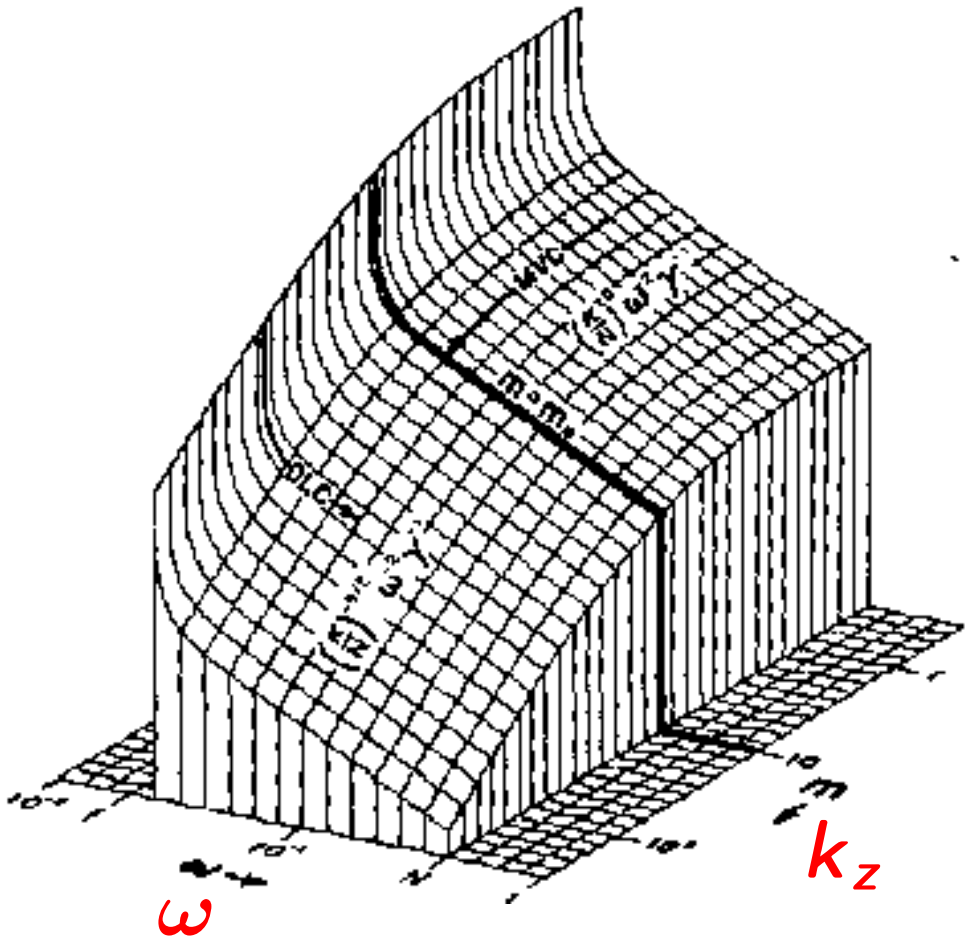


https://www.esa.int

INTERNAL WAVES

Observations interpreted as non-linear waves!

Ocean turbulence: Garrett-Munk spectrum



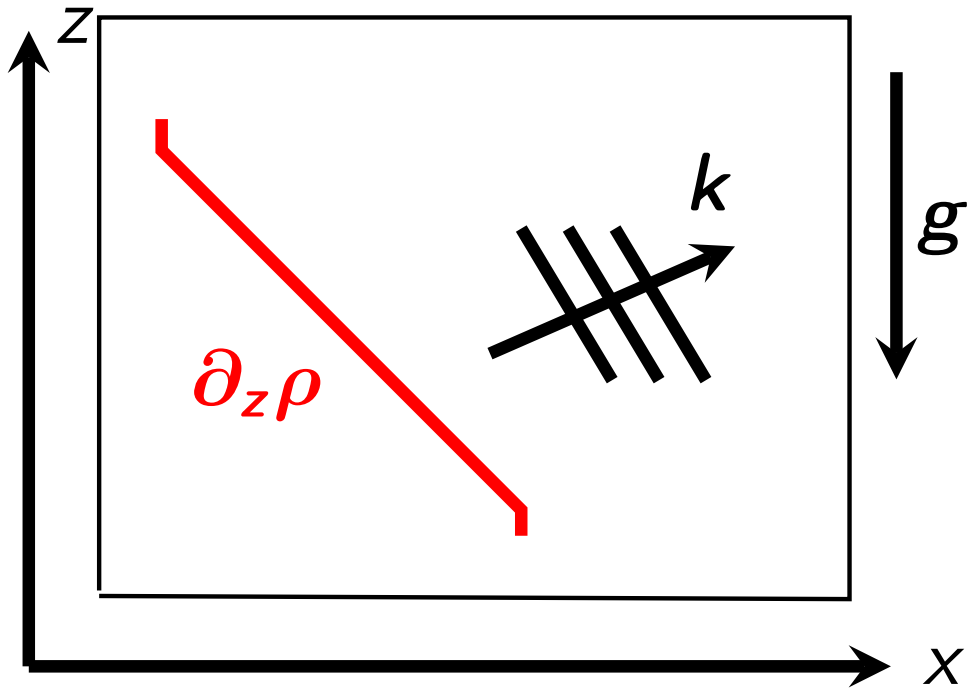
Frequency-vertical wavenumber spectra. (Garrett and Munk, 1979)

Reproduction ocean spectra with only waves?

Reduction of dimensionality...

2D stratified turbulence

- No rotation effects
- No vertical vorticity
- Wave modes + shear modes



Spectra 2D stratified turbulence $\stackrel{?}{=}$ Spectra ocean observations

Fluidsim: an open source solver

Fluidsim[1]

- Open source pseudo-spectral solver
- Language: Python/C++
- Reproductive science

Numerical parameters

- Periodic box: $L_x = 4L_z$
- Spatial resolution $n_x \times n_z$: 1920×480 (3840×960)
- Hyperviscosity ν_8
- No energy in shear modes: $E(k_x = 0, k_z) = 0$

[1] `https://bitbucket.org/fluiddyn/fluidsim`

2D Navier-Stokes equations (Boussinesq approx.)

$$D_t \xi = -\partial_x b + f_\xi + \nu_8 \nabla^8 \xi \quad \text{(vorticity)}$$
$$D_t b = N^2 u_z + f_b + \kappa_8 \nabla^8 b \quad \text{(buoyancy)}$$

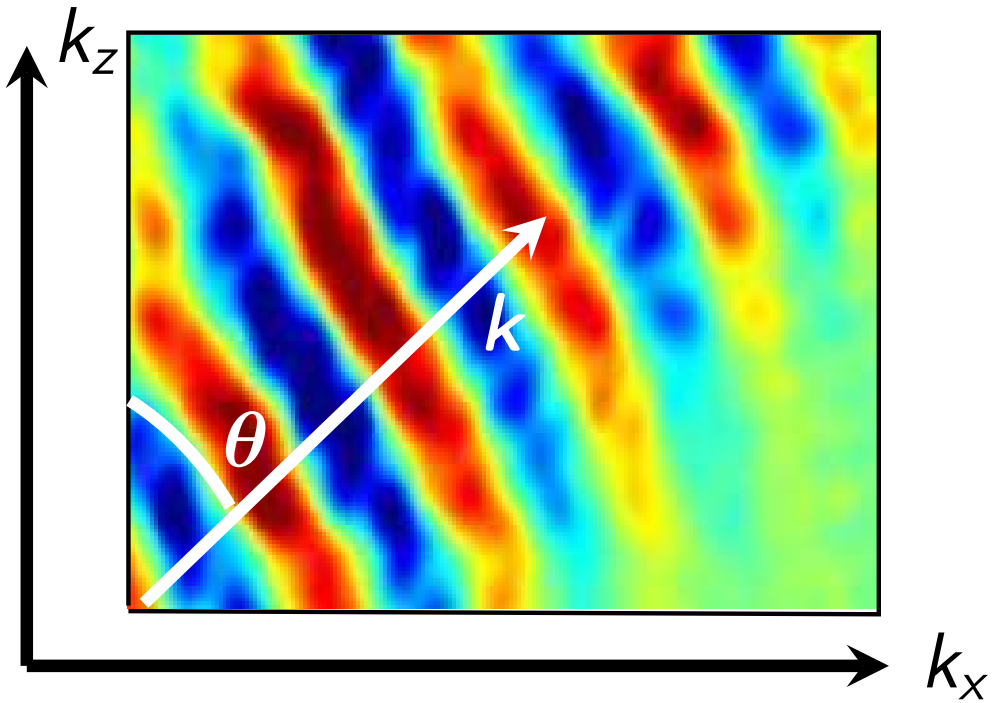
Linear dispersion relation:

$$\omega^2 = N^2 \sin^2 \theta$$

Linear eigenmodes:

$$\partial_t \hat{a}_\pm = \pm i\omega \hat{a}_\pm$$

$$\hat{a}_\pm = N^2 \hat{u}_z \pm i\omega \hat{b}$$



Background field courtesy of Lérisson, Chomaz and Ortiz (LadHyX, France)

Isotropic Vs Anisotropic forcing

Our forcing is anisotropic!

Isotropic forcing [2][3]

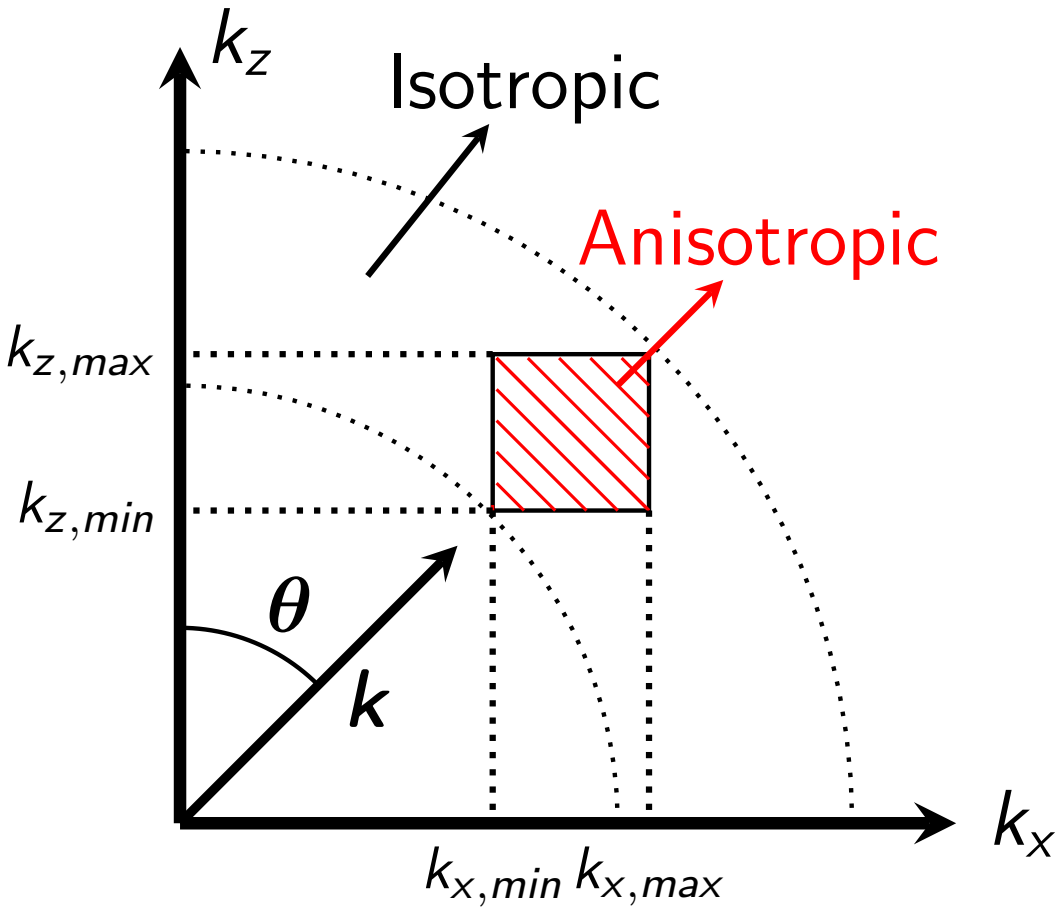
Forced waves with <u>all</u> possible frequencies.
Non-local forcing ☹

$$\omega \in [0, N]$$

Anisotropic forcing

Forced waves with <u>similar</u> frequency.
Local forcing ☺

$$\omega \sim N \sin\theta$$

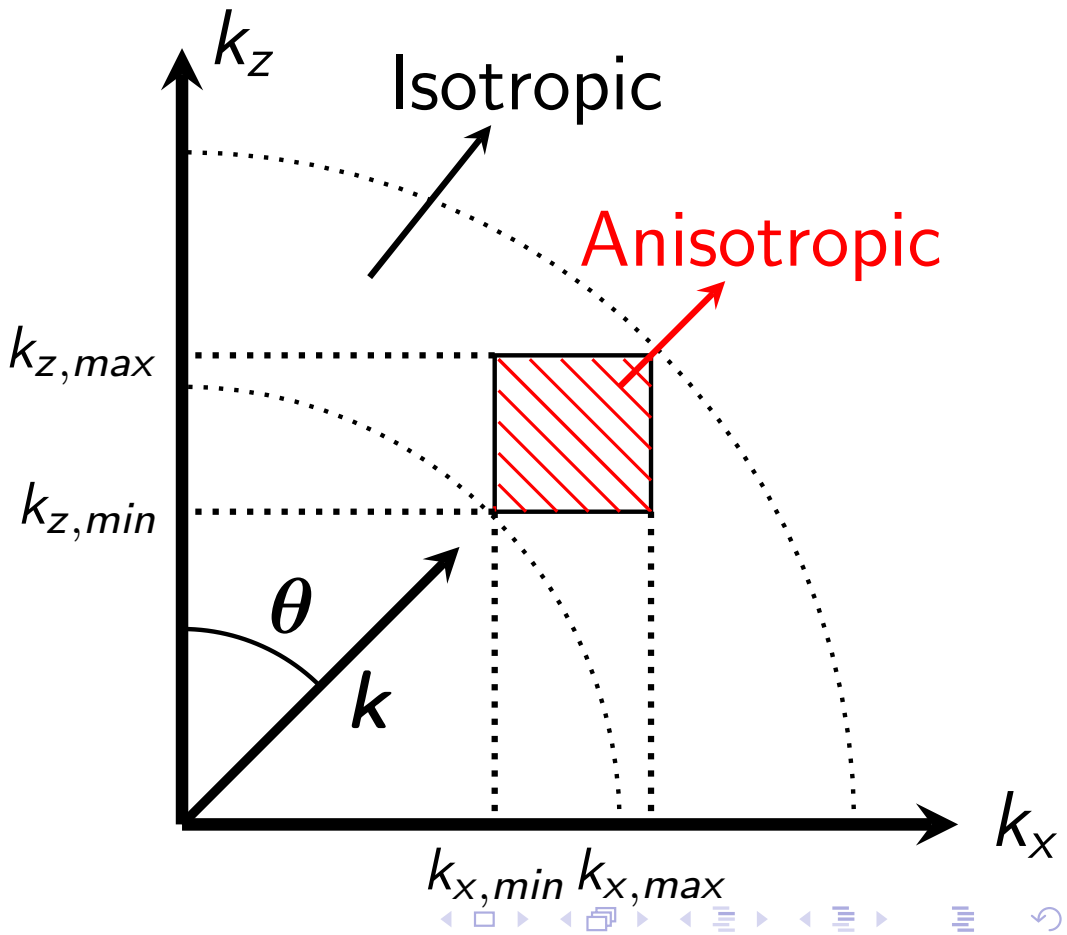


[2] G. Boffetta et al. (2011)
[3] A. Kumar et al. (2016)

Time scales

We force the linear mode $\hat{a}_+$ with an injection rate P_a

- Frequency forced waves
  $\omega_I = N \sin\theta_f$
- Amplitude forcing
  $\omega_{af} = (P_a \cdot k_f^2)^{1/7}$
- Time correlation forcing
  $\tau_{cf} = \pi/\omega_I \rightarrow$ choice!



List of simulations 1920×480

Non-dimensional parameters

$$F = \sin\theta \quad \text{(Geometry forcing)} \qquad \gamma = \frac{\omega_I}{\omega_{af}} \qquad Re = \frac{1}{\nu_8}\frac{\omega_{af}}{|\boldsymbol{k_f}|^8} \quad \text{(Reynolds)}$$

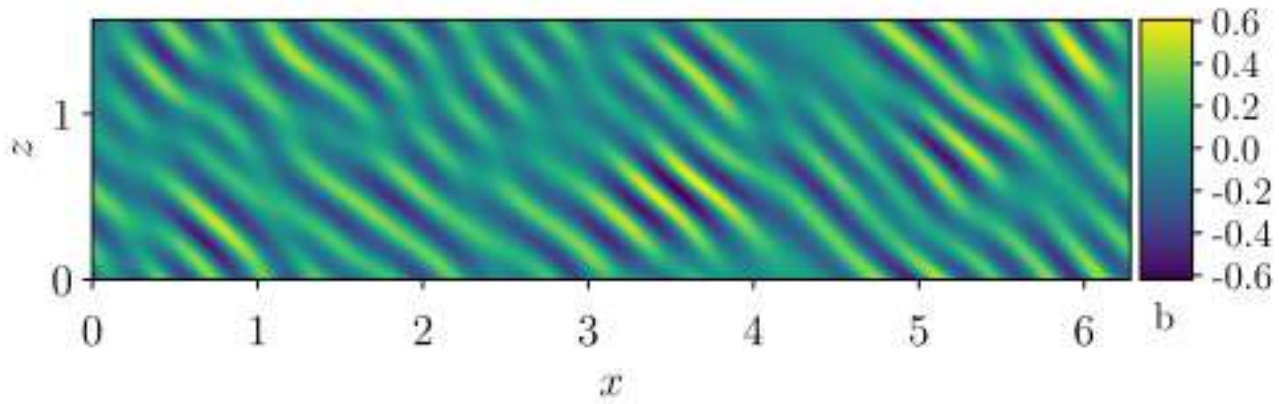
$\gamma \to \mathbf{0}$: Weakly stratified + strongly non-linear

$\gamma \to \infty$: Strongly stratified + weakly non-linear

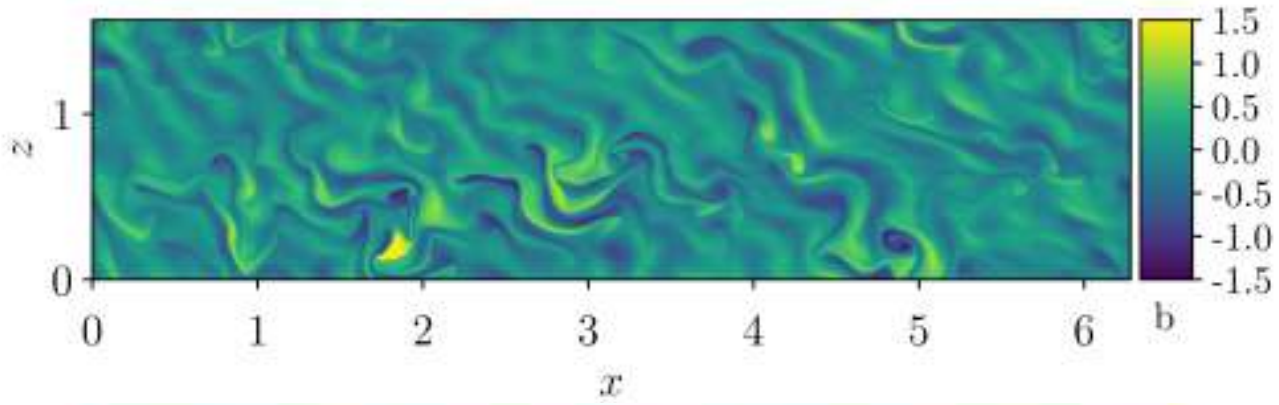
γ	$\boldsymbol{F}$
0.2	0.5
0.5	0.5
1.0	0.5

Buoyancy field $\gamma = 0.2$ (strongly non-linear)

$t/\tau_{af} = 2$

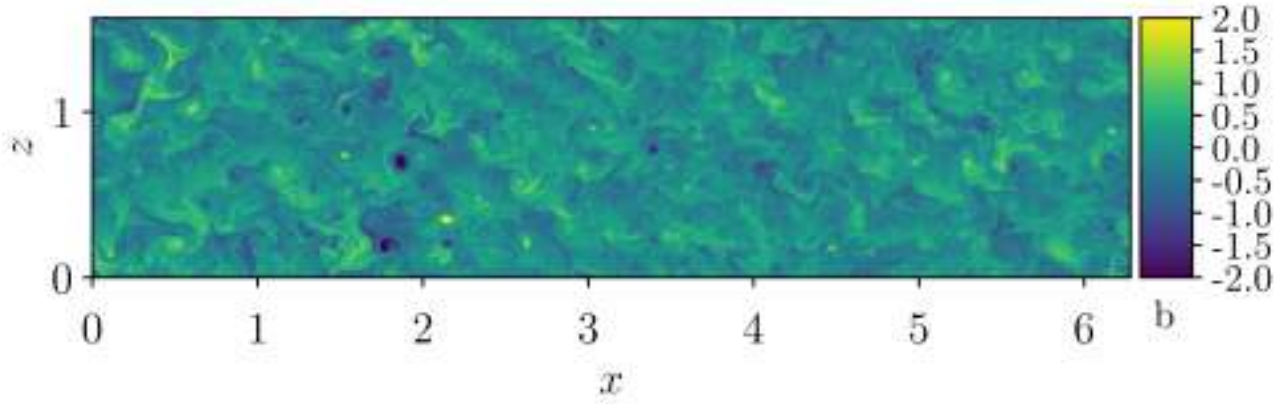


$t/\tau_{af} = 5$



$t/\tau_{af} = 1800$

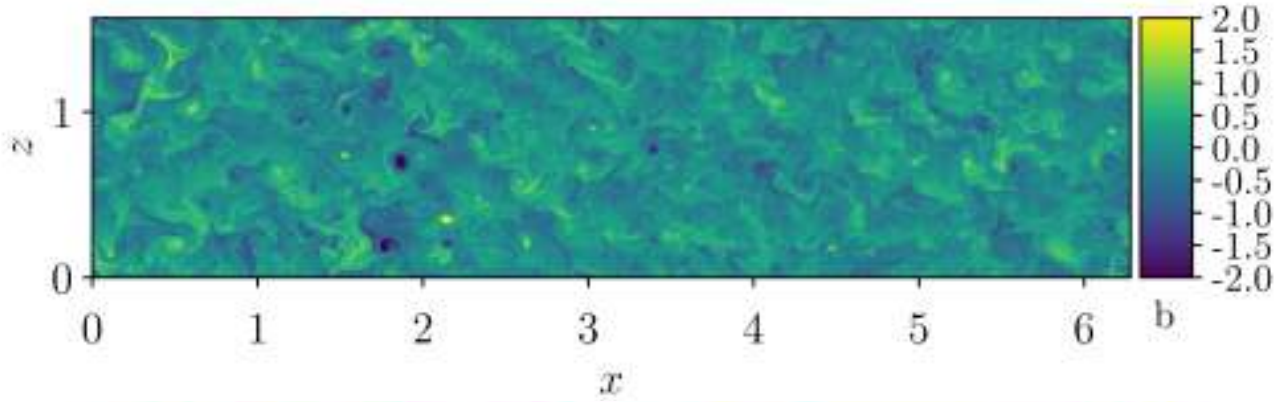
Stationary state



Buoyancy fields at stationary state

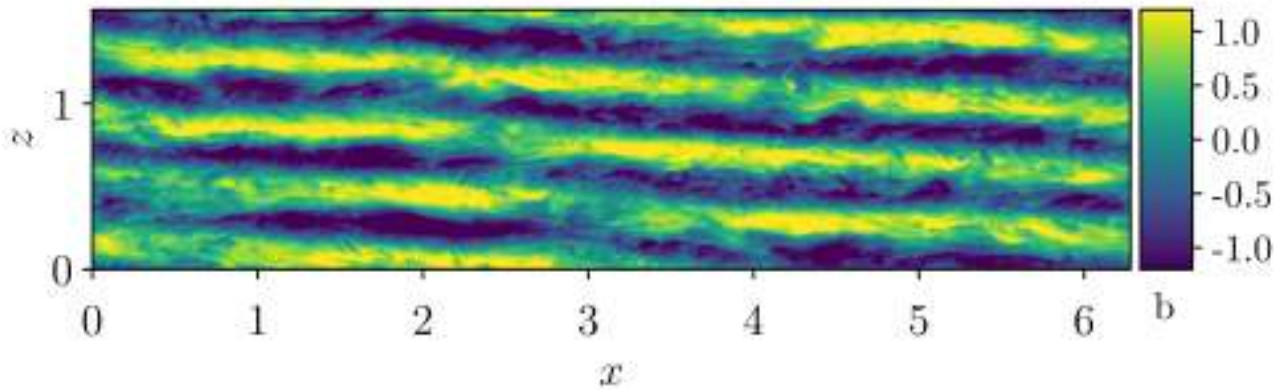
$\gamma = 0.2$

Strongly non-linear



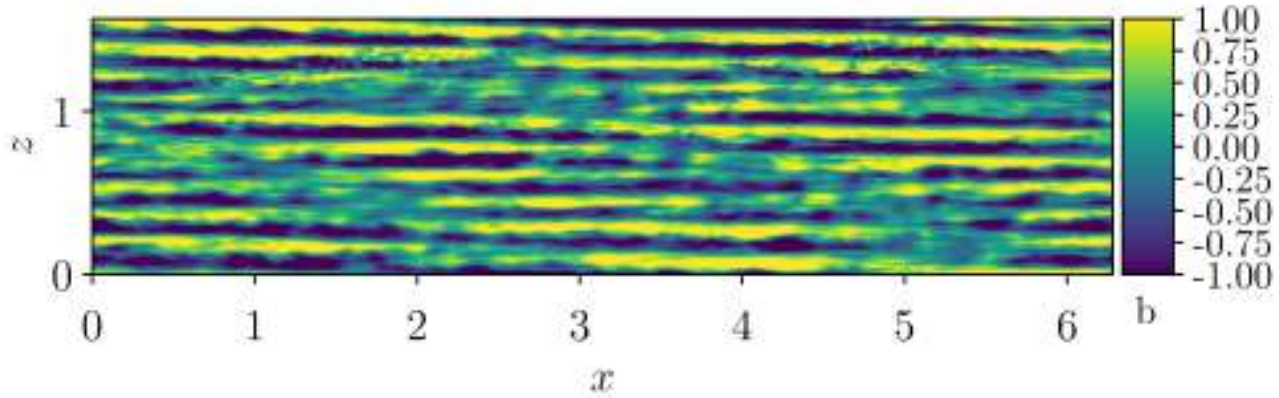
$\gamma = 0.5$

Weakly non-linear



$\gamma = 1.0$

Weakly non-linear

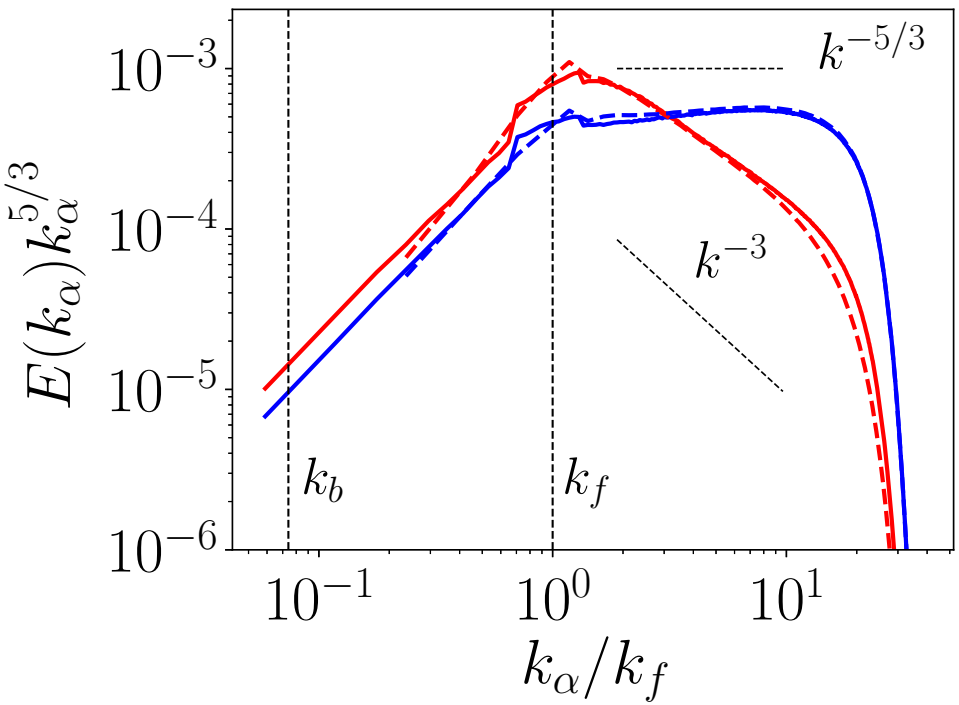


Energy spectra

Compensated vertical and horizontal spectra

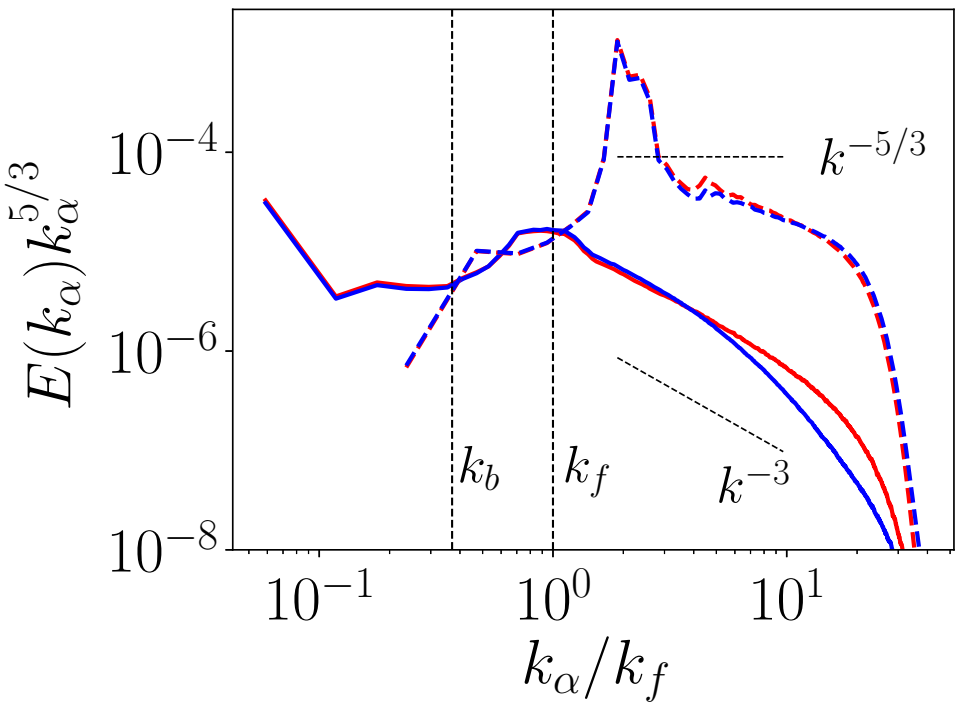
$\gamma = 0.2$

Strongly non-linear



$\gamma = 1.0$

Weakly non-linear



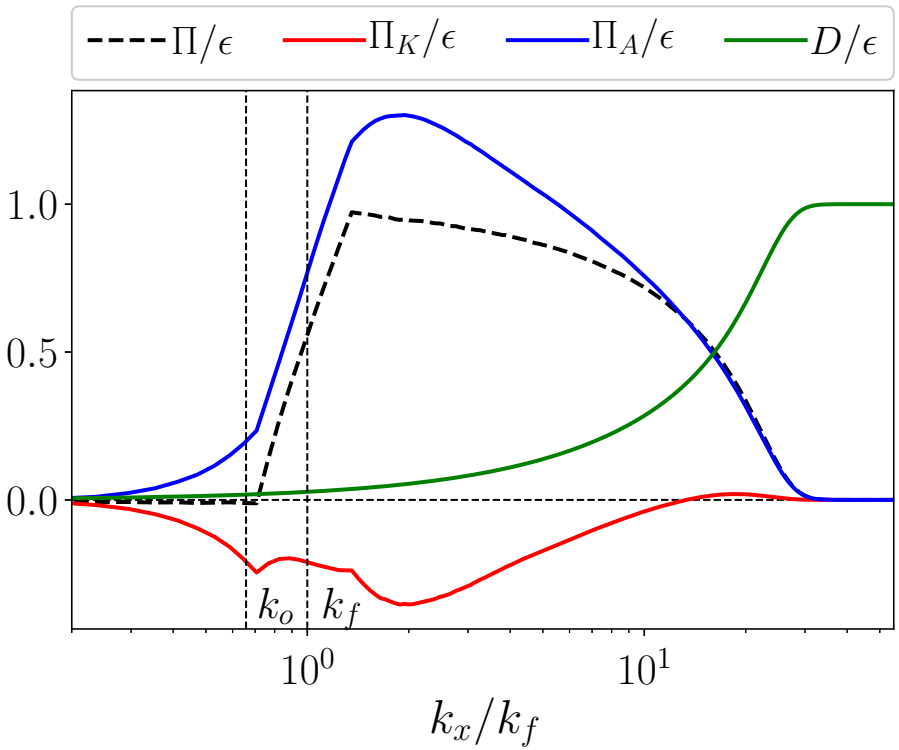
Kinetic k_x — Potential k_x

Kinetic k_z — Potential k_z

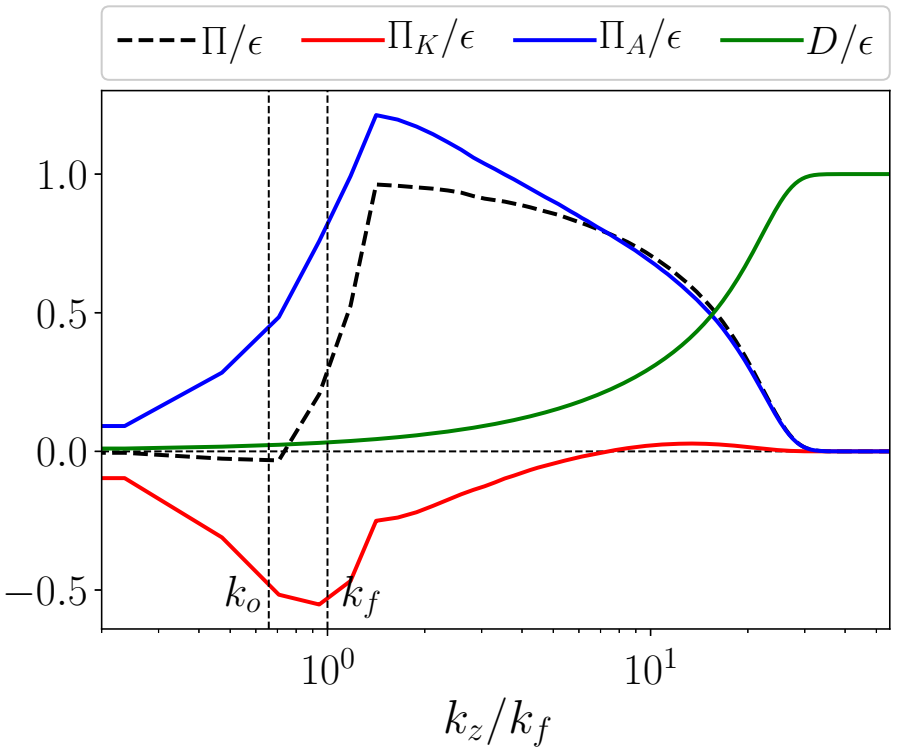
Simulation $\gamma = 0.2$ (Strongly non-linear)

Horizontal and vertical spectral energy budget

Horizontal budget



Vertical budget

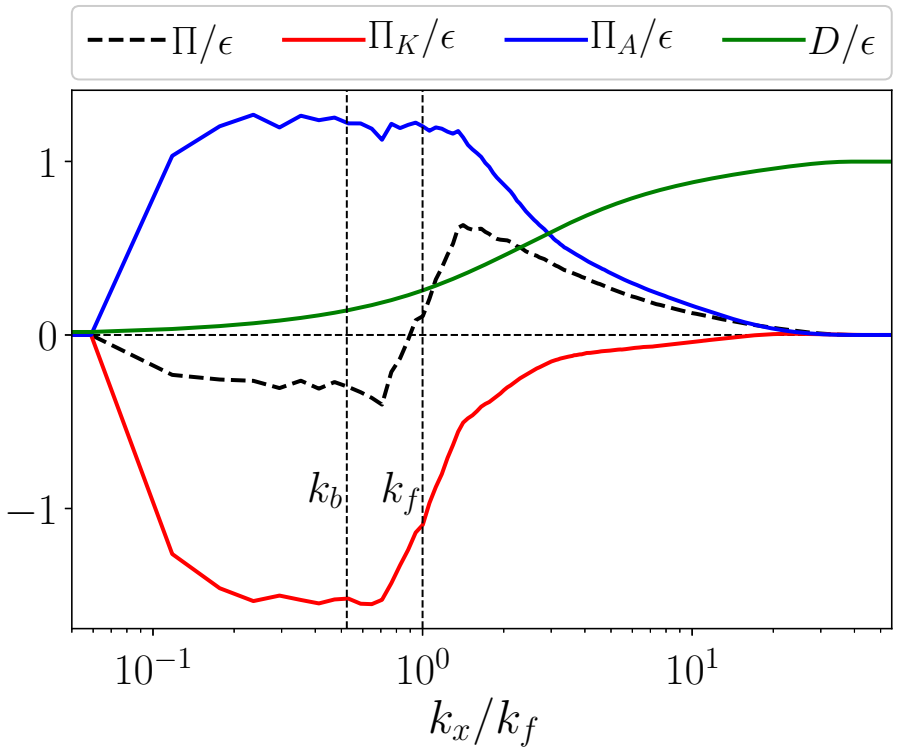


G. Boffetta et al. (2011)

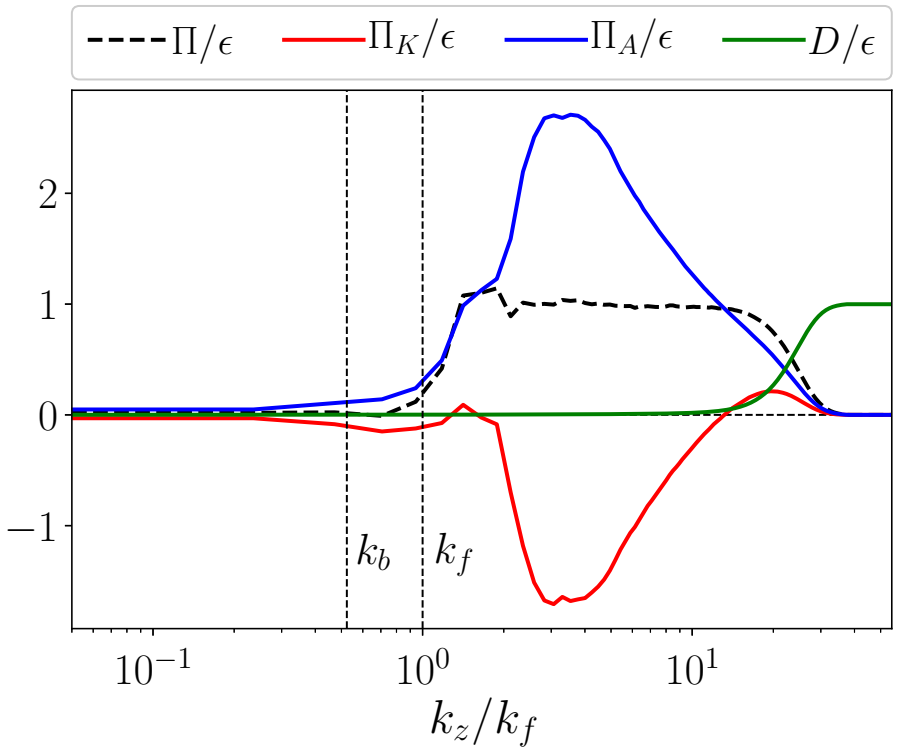
Simulation $\gamma = 1.0$ (Weakly non-linear)

Horizontal and vertical spectral energy budget

Horizontal budget



Vertical budget



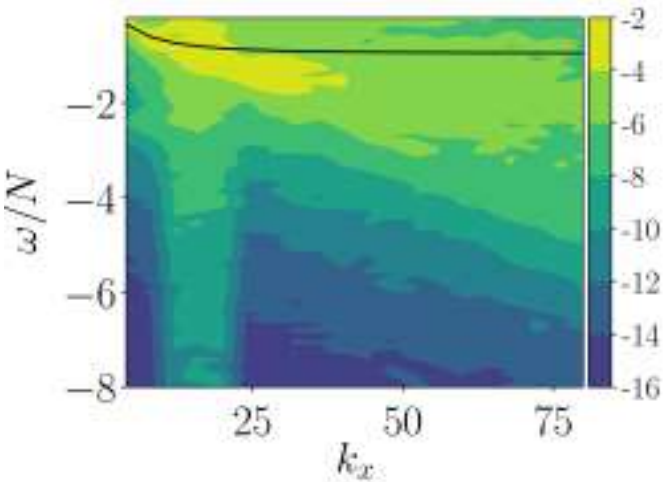
- **Loop mechanism**[5]
- **It is not a wave cascade!**

[5] G. Boffetta et al. (2011)

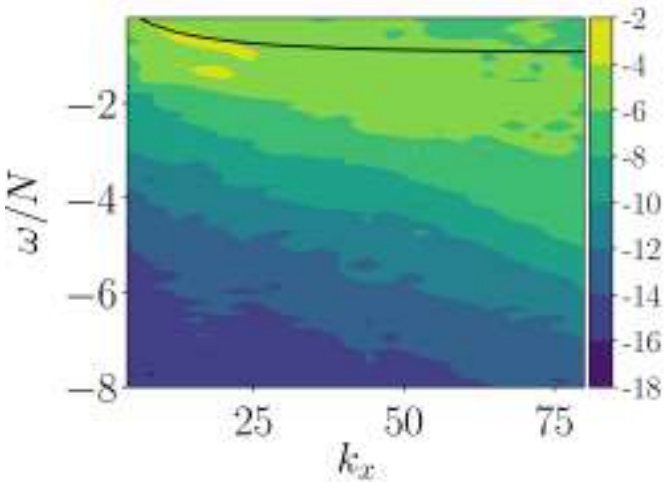
Frequency-wavenumber spectra $\gamma = 1.0$ (in progress...)

$$\omega = N \frac{k_x^2}{\sqrt{k_x^2 + k_z^2}}$$

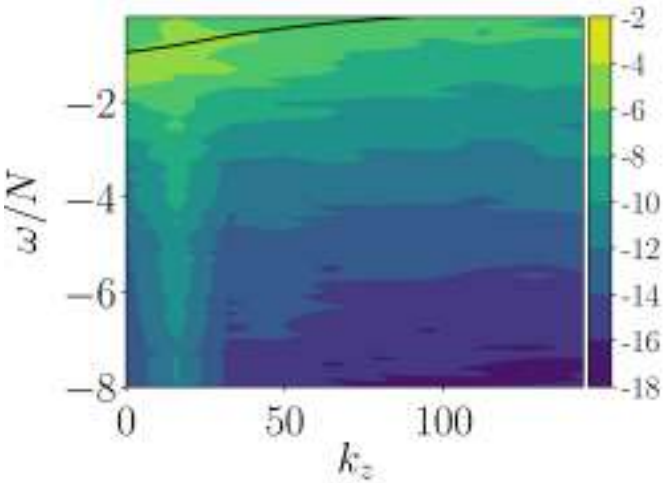
$\log_{10}(F(k_x, 16, \omega))$



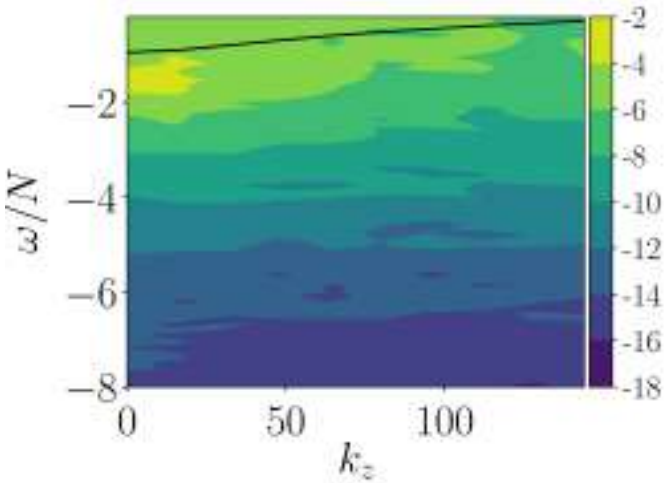
$\log_{10}(F(k_x, 32, \omega))$



$\log_{10}(F(16, k_z, \omega))$

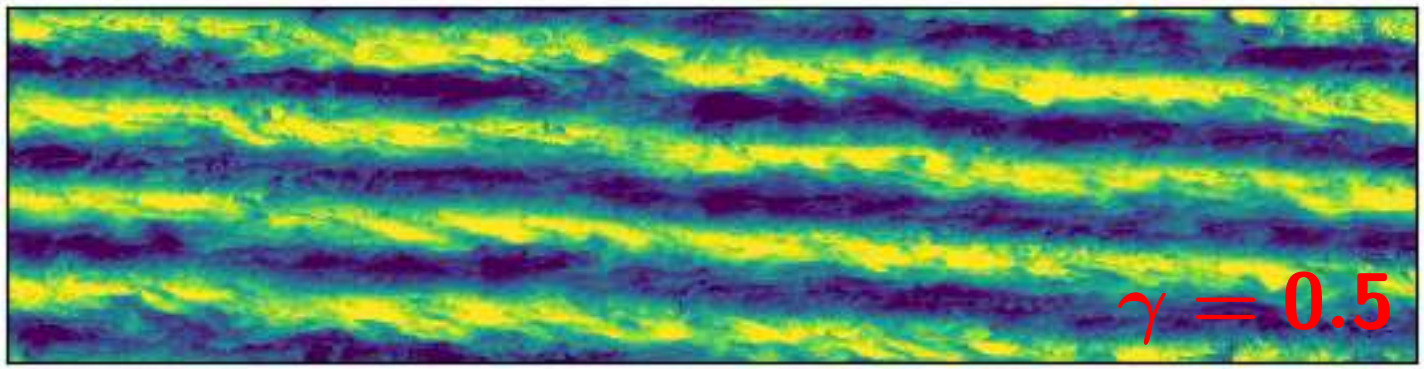


$\log_{10}(F(32, k_x, \omega))$



Conclusions

Strongly non-linear: Not layered & Isotropic dynamics & Vortices
Weakly non-linear: Layered & Anisotropic dynamics & Waves



Next steps...

- **Dynamics consistent with oceanic phenomenology?**
- **Higher resolution simulations = oceanic conditions**
- **How layers scale with the buoyancy length scale $L_b = U/N$?**
- **Further analysis of the frequency-wavenumber spectra?**

`https://bitbucket.org/fluiddyn/fluidsim`